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ARTIFICIAL STABILITY AND CONTROL OF LONGITUDINAL
MOTION OF THE F-94 AIRCRAFT

THEORETICAL INVESTIGATION

One of a Series of Reports on Artificial Stability
of Military Aircraft

Frank W. Hellenday
Cornell Aeronautical Laboratory, Inc.

October 1952

WRIGHT AIR DEVELOPMENT CENTER

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Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared by the Cornell Aeronautical Laboratory, Inc., Buffalo, New York as Cornell Aeronautical Laboratory Report No. TB-757-F-7, under USAF Contract No. AF33(038)-20659. The contract was initiated under the research and development project, identified by Research and Development Order Number R-461-1. It was administered under the direction of the Flight Research Laboratory, Wright Air Development Center with Capt. P.P. Cerussi acting as project engineer.

ABSTRACT

Variations in the longitudinal stability of the F-94 airplane can be achieved by automatic actuation of 1) the elevator proportional to $\dot{\alpha}$, α and δ_s signals to alter the short period damping and frequency, 2) the pilot's stick proportional to SF/q **and** α to change the stick force and position gradients, 3) a 0.43 ft^2 canard pitching control surface driven at 2 deg/sec. by $\overset{\circ}{u}/q$ and $\frac{\Delta q}{q}$ signals to modify the phugoid damping and period.

The operational range of the control equipment necessary to accomplish these variations in flight are determined. A specific sequence of calculation is listed for obtaining the gearing required to obtain a set of flying qualities. Transient responses determined by analog computation are presented along with a phase diagram representation of the phugoid and short period motions.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:

LESLIE B. WILLIAMS
Lt. Colonel, USAF
Chief, Flight Research Laboratory
Research Division

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DEFINITION OF SYMBOLS

c Wing mean aerodynamic chord (ft.).

c_e Elevator MAC (ft.).

C_L, C_D, C_m, C_h Lift, drag, pitching moment and hinge moment coefficients, respectively.

C_t Thrust coefficient, $C_t = \dfrac{T}{2\ qS}$

C_{m_u} Rate of change of pitching moment with non-dimensional airspeed, $dC_m/d\ (u/V)$.

$C_{m_{Du}}$ Rate of change of pitching moment with non-dimensional rate of change of airspeed,

$$dC_m/d\left[d\left(^u/_V\right)/d\left(\tfrac{t}{\tau}\right)\right]$$

$d\,C_m/d\,u$ Rate of change of pitching moment with airspeed (rad/mph)

$d\,C_m/d\,\overset{o}{u}$ Rate of change of pitching moment with rate of change of airspeed, $dC_m/d(du/dt)$(rad/mph/sec)

$d\,\delta e/d\,\alpha$ Rate of change of elevator angle with angle of attack

$d\,\delta e/d\,\overset{o}{\alpha}$ Rate of change of elevator angle with rate of change of angle of attack, (deg/deg/sec)

$d\,\epsilon/d\,\alpha$ Rate of change of downwash angle with angle of attack

$D(\ )$ $d(\ )/d(t/\tau),\ D^2 = d^2(\ )/d\ (t/\tau)^2$

$D\,u$ Rate of change of non-dimensional airspeed with non-dimensional time, $d(u/V)/d(t/\tau)$

$D\,\alpha$ Rate of change of angle of attack with non-dimensional time, $d\alpha/d\ (t/\tau)$

f Short period frequency (cps)

F_s Stick force (lbs.)

g Acceleration of gravity

H Hinge moment (ft. lbs.)

$\overset{o}{\nu}_B$ Non-dimensional pitching inertia, $\nu_B = \dfrac{g\ I_{yy}}{W\left(^c/_2\right)^2}$

l_t Tail length (ft.)

n, n_z Normal acceleration increment in g's.

P Phugoid period (sec.)

q Dynamic pressure (psf.)

q_H Horizontal tail dynamic pressure (psf.)

S Wing area (ft^2)

S_e Elevator area (ft^2)

t Time (sec.)

T Thrust (lbs.)

u Forward speed increment

u_g Forward velocity gust disturbance, (mph).

$\overset{o}{u}$ Rate of change of airspeed with time, du/dt (mph/sec.)

V Forward speed, V_i denotes indicated airspeed (mph.)

W Gross weight (lbs.)

$\propto$ Angle of attack (deg.)

$\overset{o}{\propto}$ Rate of change of $\propto$ with time, d$\propto$/dt (deg/sec.)

δ_a Auxiliary surface deflection (deg.)

δ_e Elevator deflection (deg.)

δ_f Flap deflection (deg.)

δ_s Control stick deflection (deg.)

δ_t Elevator tab deflection (deg.)

$\Delta (\ )$ Increment in ( ).

ζ_p Phugoid damping ratio.

ζ_s Short period damping ratio.

Θ Angle of pitch.

μ Relative density, $2W/\rho S \bar{g} c$

ρ Air density (lbs/ft^3)

τ Time unit, $W/\rho S \bar{g} V$

INTRODUCTION

The Flight Research Laboratory of the Wright Air Development Center has instituted a program with the Cornell Aeronautical Laboratory to obtain actual flight test data on the optimum and minimum flyable longitudinal stability and control characteristics for fighter and bomber airplanes. This type of information has recently become of great design importance with the advent of practical servo-mechanisms for the addition of artificial stability to airplanes; also, this information should be useful to those charged with the responsibility for establishing handling qualities specifications.

Two airplanes are being used for the evaluations - one, a B-26 light bomber; the second an F-94 jet fighter. The elevators of these airplanes are driven by irreversible hydraulic servos in response to control signals supplied by the pilot and signals provided by artificial stability pickups. The control sticks are driven by a second servo in response to pilot applied control force, in a manner closely simulating the natural airplane's control forces. A small auxiliary pitching surface is driven by an electric servo motor for phugoid control. By adjusting the gains of the various channels of this equipment, the following parameters of longitudinal stability and control can be varied: phugoid mode period and damping, short period mode period and damping, static elevator to trim vs. C_L and g, and static stick force vs. C_L and g. The extremes of stability and control that can easily be simulated and evaluated could not safely and economically be obtained in any other way.

Reference (1) presented the theoretical analysis on the B-26 airplane. This report contains the supplementary analysis of the artificial control necessary to provide a wide range of flying qualities for the F-94 airplane.

ANALYSIS

BASIC FLYING QUALITIES

The eight flying qualities which are considered to be basic in a pilot's evaluation of an airplane's longitudinal motion were listed in Reference (1) as follows:

1. Frequency of the "short period" mode
2. Damping of the "short period" mode
3. Period of the "phugoid" mode
4. Damping of the "phugoid" mode
5. Control deflection to trim vs. speed for 1g flight
6. Control deflection per "g" normal acceleration
7. Control force to trim vs. speed for 1g flight
8. Control force per "g"

The subsequent analysis will show how these basic quantities will be varied on the F-94 airplane.

Short Period

The short period characteristics can be determined from the formulae: (Ref. 1)

$$f = \frac{1}{2\pi\tau} \left[\frac{\mu}{\iota_\beta} (1 - \varrho_s^2)(-C_{m\alpha} - \frac{C_{L\alpha} C_{m\varrho}}{2\mu}) \right]^{\frac{1}{2}} \text{cps} \qquad (1)$$

$$100 \, \varrho_s = (100) \cdot \frac{\left(\frac{C_{L\alpha}}{2} - \frac{C_{m_{\dot{\theta}}}}{\iota_\beta} - \frac{\mu C_{m\dot{\alpha}}}{\iota_\beta} \right)}{2 \left[\frac{\mu}{\iota_\beta} \left(-C_{m\alpha} - \frac{C_{L\alpha} C_{m\varrho}}{2\mu} \right) \right]^{\frac{1}{2}}} \qquad (\% \text{ critical})$$

The normal stability derivatives of the F-94 are listed in Table I. The frequency and per cent critical damping in the short period mode are listed in Table II for five possible flight conditions.

Possible artificial control of the short period at 292 mph indicated airspeed and 20,000 ft. altitude is listed in Table III. It is noted that either $\Delta C_{m\alpha}$ or $C_{m_D}^2{}_\theta$ can be used to vary the frequency. $C_{m_D}^2{}_\theta$ changes the effective pitching inertia of the airplane, and its use yields only a limited range of frequency control, especially with large static margins. $\Delta C_{m\alpha}$ was chosen, therefore, to allow wider short period frequency variations.

The short period damping can be controlled by either $\Delta C_{m_{D\alpha}}$ or ΔC_{m_q} as seen in Table III. ΔC_{m_q}, however, affects the phugoid mode and the maneuvering stability. $\Delta C_{m_{D\alpha}}$ was chosen, therefore, in order to isolate the effects of each variable on the eight basic flying qualities. It should be noted that either $\Delta C_{m\alpha}$

or $\Delta c_{m_{D\alpha}}$ changes both the frequency and the damping of the short period mode. Thus, a sequence of settings for these artificial controls has been established as follows:

1. Choose desired f and ζ_s of short period.
2. Calculate $d\delta e/d\alpha$ or $\Delta c_{m_{\delta e}}$ required from:
(Ref. 1)

$$\frac{d\delta e}{d\alpha} = \frac{\Delta C_{m\alpha}}{C_{m\delta e}} = \frac{1}{C_{m\delta e}}\left[-C_{m\alpha} - \frac{C_{L\alpha}C_{mq}}{2\mu} - \frac{(2\pi\tau f)^2 i_B}{\mu(1-\zeta_s^2)}\right] \quad (2)$$

3. Calculate $d\delta e/d\overset{\circ}{\alpha}$ or $\Delta c_{m_{D\alpha}}$ required from:

$$\frac{d\delta e}{d\overset{\circ}{\alpha}} = \frac{\tau\Delta C_{m_{D\alpha}}}{C_{m\delta e}} = \frac{\tau i_B}{C_{m\delta e}\mu}\left\{\frac{C_{L\alpha}}{2} - \frac{C_{mq}}{i_B} - \frac{\mu}{i_B}C_{m_{D\alpha}}\right. \quad (3)$$

$$\left. -2\zeta_s\left[\frac{\mu}{i_B}\left(-C_{m\alpha} - C_{m\delta e}\frac{d\delta e}{d\alpha} - \frac{C_{L\alpha}C_{mq}}{2\mu}\right)\right]^{\frac{1}{2}}\right\}$$

$$(\text{rad/rad/sec})$$

Phugoid

The phugoid characteristics of the normal F-94 airplane can be determined from the formulae:
(Ref. 1)

$$P = \frac{2\pi\tau}{C_L}\left[\frac{2\left(C_{m\alpha} + \frac{C_{mq}C_{L\alpha}}{2\mu}\right)}{C_{m\alpha}(1-\zeta_p^2)}\right]^{\frac{1}{2}} \quad \text{sec.}$$

$$100\,\zeta_p = 100\left(\frac{\Lambda}{2C_{m\alpha}}\right)^{\frac{1}{2}}\left\{\frac{C_D}{C_L} + \frac{C_L}{2\mu\Lambda}\left[\mu C_{m_{D\alpha}} + C_{mq}\left(1 - \frac{C_{0\alpha}}{C_L}\right)\right]\right. \quad (4)$$

$$\left. + \frac{i_B C_L C_{m\alpha}}{\mu 2\Lambda^2}\left(\frac{C_{L\alpha}}{2} - \frac{C_{mq}}{i_B} - \frac{\mu}{i_B}C_{m_{D\alpha}}\right)\right\} \quad (\% \text{ critical})$$

$$\Lambda = C_{m\alpha} + \frac{C_{L\alpha}C_{mq}}{2\mu}$$

If the static margin is greater than 10%, the phugoid period and damping are closely approximated by the formulae:

$$P = .202\ V_{mph} \quad (\text{sec.})$$

$$100\,\zeta_p = 70.7\frac{C_D}{C_L} \quad (\% \text{ critical})$$

- 2 -

The normal phugoid characteristics of the F-94 should be calculated from the former set of equations as the static margin is close to 5.5% for the usual flight conditions. Table II lists the period and damping for five possible flight configurations.

Possible artificial control of the phugoid at 292 mph indicated airspeed and 20,000 ft. altitude is listed in Table IV. The purpose of this program is to provide a wide range of control over the flying qualities. Variations in thrust (or C_t) would not provide this extreme range in controlling large amplitude disturbances. This can be seen for example, if a C_{x_u} = -.17 is assumed to add

50% critical damping to the phugoid. If the airspeed varied by 20 mph ind. a change in thrust of 1220 lbs would be needed, to achieve this damping. Thus, it was not considered practical in this case to utilize thrust variations for phugoid control.

Pitching moment derivatives proportional to u or D^2u can be used to vary the phugoid period. $C_{m_{D^2u}}$, however, affects the short period mode, so that C_{m_u} has been chosen for artificial control.

Either $C_{m_{Du}}$ or C_{m_θ} will control the phugoid damping. Since C_{m_θ} would also vary the static stability of the aircraft, $C_{m_{Du}}$ was chosen. It should be noted that both $C_{m_{Du}}$ and C_{m_u} should be used simultaneously to achieve a desired period and damping, since either derivative affects both period and damping.

Auxiliary Surface

It is noted in Table IV that a dS_e/du of -.0059 deg/mph ind. would be required to reduce the phugoid period to 62.8 sec. This gearing would demand a positioning accuracy of 0.0059 deg. elevator if the threshold of the instrumentation is assumed to be 1 mph airspeed variation. Therefore, the elevator can not be used to provide the low pitching moments required to control the phugoid. The design of a small pitching control auxiliary surface is now considered.

The maximum pitching moment necessary will in general determine the dimensions of the auxiliary surface. In this case, however, the design was determined by calculating the minimum moment desired and then multiplying this value by 100 to yield a practical gearing range for instrumentation. The minimum C_m would be required to add 10% critical damping to the phugoid at the V_i = 292 mph, H = 20,000 ft., dC_m/dC_L = -.055 condition when the disturbance is of 1 mph amplitude.

$$C_{m_{min.}} = C_{m_{Du}}Du$$

$$C_{m_{Du}} = \frac{4}{100} \Delta C_s \left[2dC_m/dC_L \left(dC_m/dC_L + C_{m_q}/2\mu \right) \right]^{\frac{1}{2}}$$

(Ref. 1)

$$C_{m_{Du}} = 4(.10)\left[2(-.055)(-.0661)\right]^{\frac{1}{2}} = .034$$

$$Du = \frac{\Delta V}{V}\left(\frac{2\pi\tau}{P}\right)\frac{1}{292}\frac{2\pi(2.4)}{88.6} = .000583$$

$$C_{m_{min.}} = .034(.000583) = .00198 \times 10^{-2}$$

If a 100 to 1 ratio is assumed for the sensitivity range, a design C_m of .002 will be determined.

A deflection range of $\pm$ 10 deg. was assumed to give a $C_{m_{\delta_a}}$ of .0002 1/deg. The dimensions of the surface were determined from the formula:

$$S_a = \frac{C_{m_{\delta_a}}}{C_{L_\alpha}\frac{\ell_t}{\bar{c}}\frac{1}{S}}$$

For an assumed aspect ratio of 2.87 for each side of the surface, C_{L_α} will be 3.06. The surface will be mounted in the nose 14 ft. from the c.g. of the airplane. Thus:

$$S_a = \frac{.0002\,(57.3)}{3.06\left(\frac{14}{6.72}\right)\frac{1}{237.6}} = .43\ ft^2$$

The dimensions of the surface are then:

span on one side = 9 3/8 in.
chord = 3 1/4 in.

The formulae for $d\delta_a/du$ and $d\delta_a/d\overset{\circ}{u}$ in terms of the phugoid damping and period are:

$$\frac{d\delta_a}{du} = \frac{C_{m_u}}{V_i C_{m_{\delta_a}}} = \frac{2C_L}{V_i C_{L_\alpha} C_{m_{\delta_a}}}\left[C_{m\alpha_T} - \left(\frac{2\pi\tau}{PC_L}\right)^2 \frac{2\lambda_T}{(1-\zeta_p^2)}\right] \quad (5a)$$

$$\text{deg/mph ind.}$$

where $C_{m\alpha_T} = C_{m\alpha} + C_{m_{\delta_e}}\,d\delta e/d\alpha$

$$\lambda_T = C_{m\alpha_T} + \frac{C_{L\alpha}\,C_{m_u}}{2\mu}$$

$$\frac{d\delta a}{du} = \frac{\tau C_{m_{Du}}}{V_i C_{m_{\delta a}}} = \frac{4\mathcal{Z}\Delta\tau}{V_i C_{m_{\delta a}} C_{L_\alpha}}\left\{\frac{C_D}{C_L} + \frac{C_L}{2\mu\Delta\tau}\left[\mu C_{m_{D\alpha_T}} + C_{m_g}\left(1 - \frac{C_{D\alpha}}{C_L}\right)\right]\right.$$

$$\left. + \phi\frac{C_L}{2}\frac{i_B}{\Delta\tau^2\mu}\left[\frac{C_{L_\alpha}}{2} - \frac{C_{m_g}}{\mu_B} - \frac{\mu}{\mu_B}C_{m_{D\alpha_T}}\right] - \zeta_P\left[\frac{2\phi}{\lambda_\tau}\right]^{\frac{1}{2}}\right\} \begin{array}{l}\text{deg/mph}\\ \text{ind/sec.}\end{array}$$

(6a)

where $\quad C_{m_{D\alpha_T}} = C_{m_{D\alpha}} + C_{m_{\delta e}}\dfrac{d\delta e/d\alpha^o}{\tau}$

and $\quad \phi = C_{m_{\alpha_T}} - \dfrac{C_{L_\alpha}}{2C_L}V_i C_{m_{\delta a}}\dfrac{d\delta a}{du}$

The sequence of calculations is important. Steps (1), (2) and (3) should be completed first for the short period analysis and then:

4. Choose ζ_P and P of the phugoid desired.

5. Calculate $d\delta a/du$ or C_{m_u} as above.

6. Calculate $d\delta a/d\overset{o}{u}$ or $C_{m_{Du}}$ as above.

It was noted in Reference (1) that the signals $\Delta\left(\frac{1}{q}\right)$ and $\overset{o}{u}/q$ would be provided instead of u and $\overset{o}{u}$ in order to vary the phugoid in a "natural" way, i.e. independent of speed. The signal $\Delta\frac{1}{q}$ was provided by a dividing network as described in Reference (2) which allowed the full range of q while discerning a 1 mph variation. The q range of the F-94 is greater than that of the B-26 and with the use of a larger range gage, the discrimination can no longer be held near 1 mph (or 1 psf.).

A $\Delta q/q$ signal will be used for the F-94 instead. This signal can be measured accurately by differential pressure gages on total and static lines and with a dividing network as before. The relations between $\Delta\frac{1}{q}$ and $\Delta q/q$ can be noted as:

$$\Delta\left(\frac{1}{q}\right) = \left(\frac{1}{q}\right)_1 - \left(\frac{1}{q}\right)_0 = \frac{1}{q_1} - \frac{1}{q_0} = \frac{q_0 - q_1}{q_1 q_0} = -\frac{1}{q_0}\left(\frac{\Delta q}{q}\right)$$

Thus it is apparent that $\Delta q/q$ is proportional to $\Delta(1/q)$ and that the $\Delta q/q$ signal will be inversely proportional to the equilibrium dynamic pressure. This signal must then be adjusted for the trim speed of the test.

The auxiliary surface will then be controlled by signals from $\Delta q/q$ and $\overset{o}{u}/q$.

The formulae for $d\delta_a/d\frac{\Delta g}{g}$ and $d\delta_a/d\frac{a}{g}$ are:

$$\frac{d\delta_a}{d\frac{\Delta g}{g}} = \frac{V_i}{2} \qquad \frac{d\delta_a}{du} = \frac{C_{mu}}{2C_{m\delta_a}} \tag{5b}$$

$$\frac{d\delta_a}{d\frac{a}{g}} = q \qquad \frac{d\delta_a}{du} = \frac{\rho \tau^2 C_{m_{pu}}}{V_i C_{m\delta_a}} \tag{6b}$$

Control Stick Parameters

The control stick parameters of the normal F-94 airplane were calculated from the formulae of Ref. (1):

$$\frac{d\delta_e}{dV_i} = \frac{2 C_L \, dC_m/dC_L}{C_{m\delta_e} V_i} \qquad \text{deg/mph ind.}$$

$$\frac{dF_s}{dV_i} = -\frac{2V_i}{V_{i\,\text{TRIM}}} \frac{\partial F_s}{\partial H} \frac{S_e c_e W/S}{C_{m\delta_e}} \frac{\rho H}{g} C_{h\delta_e} \left[\frac{dC_m}{dC_L} - C_{m\delta_e} \frac{C_{h\alpha_t}}{C_{h\delta_e}} \frac{(1-\frac{d\epsilon}{d\alpha})}{C_{L\alpha}}\right]$$

$$\text{lbs/mph ind.}$$

where F_s is positive for push forward

$$\tag{7}$$

$$\frac{d\delta_e}{dn} = \frac{C_L}{C_{m\delta_e}} \left[\frac{dC_m}{dC_L} + \frac{dC_{m_q}}{2\mu}\right] \qquad \text{deg/g}$$

$$\frac{dF_s}{dn} = \frac{\partial F_s}{\partial H} S_e c_e \cdot \frac{\rho H}{g} \left\{ \frac{x W}{S} \frac{\ell_t}{c} \frac{1}{\mu} \left[C_{h\alpha_t} - \frac{C_{m_q} C_{h\delta_e}}{2\frac{\ell_t}{c}C_{m\delta_e}}\right] \right.$$

$$\left. - \frac{W/S}{C_{m\delta_e}} C_{h\delta_e} \left[\frac{dC_m}{dC_L} - C_{m\delta_e} \frac{C_{h\alpha_t}}{C_{h\delta_e}} \frac{(1-\frac{d\epsilon}{d\alpha})}{C_{L\alpha}}\right] \right\} \quad \text{lbs/g}$$

where n is positive for push down
  x = 1 for push downs
  x = $(1 + \frac{1}{\mu})$ for turns

The control stick parameters of the normal F-94 are listed in Table V. In order to provide artificial control the quantities $d\delta_e/d\delta_s$, $d\delta_s/d\alpha$ and $d\delta_s/dF_s/g$ will be provided. Since the stick is no longer connected to the elevator surface mechanically, relations can be determined to find the proper value of these three variables for desired stick gradients. It is again important to sequence this calculation as shown below:

7. Choose $d\delta_e/dV_i$ or $d\delta_e/dn$ desired.

8a. Calculate $d\delta_e/d\delta_s$ (if $d\delta_e/dV_i$ is chosen) from:

$$\frac{d\delta_e}{d\delta_s} = \frac{C_{m_{\delta_s}}}{C_{m_{\delta_e}}} = \frac{2C_L\left[\frac{C_{m_{\alpha_T}}}{C_{L_\alpha}} - \frac{C_{m_u}}{2C_L}\right]}{V_i\,C_{m_{\delta_e}}\,d\delta_e/dV_u} \qquad \text{deg/deg} \quad (8a)$$

or 8b. Calculate $d\delta_e/d\delta_s$ (if $d\delta_e/dn$ is chosen) from:

$$\frac{d\delta_e}{d\delta_s} = \frac{C_{m_{\delta_s}}}{C_{m_{\delta_e}}} = \frac{C_L\left[\frac{C_{m_{\alpha_T}}}{C_{L_\alpha}} - \frac{C_{m_u}}{2C_L} + \frac{C_{m_e}}{2\mu}\right]}{C_{m_{\delta_e}}\,d\delta_e/dn} \qquad \text{deg/deg} \quad (8b)$$

9. Choose dF_s/dV_i and dF_s/dn desired

10. Calculate $d\delta_s/d\alpha$ from

$$\frac{d\delta_s}{d\alpha} = -\frac{C_{l_{\alpha_t}}(1-\frac{d\epsilon}{d\alpha})}{C_{h_{\delta_s}}} = \frac{-\frac{\partial F_s/\partial n}{\partial F_s/\partial V_i}C_{m_{\alpha_T}} + \eta_T\frac{V_i}{2}}{C_{m_{\delta_s}}\left[\frac{\partial F_s/\partial n}{\partial F_s/\partial V_i} - \frac{V_i}{2}\left(1 + \frac{\frac{g_c C_{L_\alpha}}{2}}{\mu}(1-\frac{d\epsilon}{d\alpha})\right)\right]}$$

(at trim speed in 1g flight) $\qquad$ deg/deg $\quad$ (10)

11. Calculate $dF_s/q/d\delta_s$ from

$$\frac{dF_s/q}{d\delta_s} = \frac{\partial F_s}{\partial H}\,\frac{2H}{q}\,S_e c_e\,C_{h_{\delta_s}}$$

$$\frac{dF_s/q}{d\delta_s} = \frac{\partial F_s}{\partial V_i}\,\frac{V_i}{2}\,\frac{C_{m_{\delta_s}}}{W/S}\,C_{L_\alpha}\left[\frac{1}{C_{m_{\alpha_T}}+C_{m_{\delta_s}}\frac{d\delta_s}{d\alpha}}\right] \quad (11)$$

(ft^2/deg)

PREDICTING FLYING QUALITIES

The steps to predict the flying qualities are listed below as a summary of the preceding analysis:

1. Choose f and ζ_{s} of short period desired.
2. Find $d\delta_e/d\alpha$ from Equation (2).
3. With this $d\delta_e/d\alpha$, use Equation (3) to determine $d\delta_e/d\dot{\alpha}$ or $\Delta C_{m_{D\alpha}}$.
4. Choose P and ζ_p of phugoid desired.
5. With $d\delta_e/d\alpha$ and $d\delta_e/d\dot{\alpha}$ found above, use Equation (5a) and (5b) to determine $d\delta_u/d\frac{u}{q}$ or C_{m_u}.
6. With $d\delta_u/d\alpha$, $d\delta_e/d\dot{\alpha}$ and $d\delta_u/du$ determined use Equation (6a) and (6b) to find $d\delta_u/du/q$ or $C_{m_{Du}}$.

7. Choose either $d\delta e/dV_i$ or $d\delta e/dn$.

8a. With ΔC_{m_α} and C_{m_u} known, use Equation (8a) to find $C_{m_{\delta_s}}$ or $d\delta e/d\delta_s$ if $d\delta e/dV_i$ is chosen.

or 8b. With ΔC_{m_α} and C_{m_u} known, use Equation (8b) to find $C_{m_{\delta_s}}$ or $d\delta e/d\delta_s$ if $d\delta e/dn$ is chosen.

9. Choose dF_s/dV_i and dF_s/dn desired.

10. With ΔC_{m_α} known, use Equation (10) to determine $d\delta_s/d\alpha$ or $C_{h_{\alpha_t}}(1-\frac{d\epsilon}{d\alpha})/C_{h_{\delta_s}}$.

11. With ΔC_{m_α}, $C_{m_{\delta_s}}$ and $d\delta_s/d\alpha$ known, use Equation (11) to determine $C_{h_{\delta_s}}$ or $\frac{dF_s/q}{d\delta_s}$.

RANGE OF VALUES

The gearings $d\delta e/d\alpha$, $d\delta e/d\overset{\circ}{\alpha}$ $d\delta\omega/d\frac{\Delta v}{v}$, $d\delta\omega/d\overset{\circ}{u}/v$, $d\delta e/d\delta_s$, $d\delta_s/d\alpha$ and $dF_s/q/d\delta_s$ have been chosen for automatic control. The ranges of values of these parameters will next be determined.

Elevator Servo

Modifications to the normal F-94 airplane will be made to actuate the elevator with no mechanical connection to the pilot's stick. It should be noted here that a stick will be added in the F-94 aft cockpit to allow the stand-by pilot to maintain direct mechanical control of the elevator surface.

An electronic-hydraulic servo system similar to that described in Reference (2) will be used to actuate the elevator surface. Full elevator travel of 54 deg. will be provided with 6 in. servo linear travel. The design resolution of the servo is 1/6 of 1% or .01 in which is equivalent to .09 deg. The signals to the elevator servo are shown below:
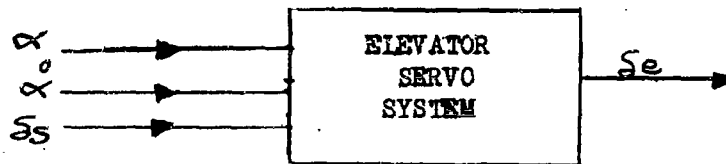
The equipment will be designed for a short period frequency extremes of 0.1 to 1.0 cps and for an angle of attack of from ± 0.07 to ± 10 deg.

The signal $d\delta e/d\alpha$ will be used to provide a minimum 1% static margin change. Thus, from Equation (2):

$$\frac{d\delta e}{d\alpha} = \frac{C_{L_\alpha} \Delta \frac{dC_m}{dC_L}}{C_{m_{\delta_e}}} = \pm \frac{5.9 \,(.01)}{.945} = \pm .0625 \text{ deg/deg}$$

The requirement that the frequency of the short period be doubled at the high static margin of 11%, determines the maximum positive gain of $d\delta e/d\alpha$ as:

$$\frac{d\delta e}{d\alpha} = 3\frac{C_{m_\alpha}}{C_{m_{\delta e}}} = \frac{3(5.9)(-.11)}{-.945} = 2.06 \text{ deg/deg}$$

In order to halve the frequency at this condition:

$$\frac{d\delta e}{d\alpha} = -\frac{3}{4}\frac{C_{m_\alpha}}{C_{m_{\delta e}}} = -.51 \text{ deg/deg}$$

The signal $d\delta e/d\alpha$ will provide a minimum 10% critical damping change in the short period at $V_i = 240$ mph ind. and 2% static margin.
From Equation (3):

$$\frac{d\delta e}{d\dot\alpha} = -\frac{2\tau\Delta\zeta}{C_{m_{\delta e}}}\left[\frac{i_B C_{L_\alpha}}{\mu}\left(-\frac{dC_m}{dC_L} - \frac{C_{m_\alpha}}{2\mu}\right)\right]^{\frac{1}{2}}$$

$$= \pm\frac{2(2.40).10}{.945}\left[\frac{5.54}{419}(5.9)(.02+.0111)\right]^{\frac{1}{2}} = \pm .025 \text{ deg/deg/sec.}$$

The maximum sensitivity is necessary to increase the short period damping to 140% critical at $V_i = 292$ mph ind. and 10% static margin. Thus, $\Delta\zeta = 101.8\%$ critical and $d\delta e/d\dot\alpha$ is:

$$\frac{d\delta e}{d\dot\alpha} = \frac{2(2.4)1.018}{.945}\left[\frac{5.54}{419}(5.9)(.10+.0111)\right]^{\frac{1}{2}} = .48 \text{ deg/deg/sec}$$

In order to bring the damping to zero:

$$\frac{d\delta e}{d\dot\alpha} = \frac{2(2.4)}{.945}(-.495)\left[.093\right] = -.23 \text{ deg/deg/sec}$$

The signal $d\delta e/d\delta s$ varies the stick position gradient with airspeed and normal acceleration. It was desired to vary $d\delta e/dV_i$ and $d\delta e/dn$ from slightly unstable to five times their normal stable value. $d\delta e/d\delta s$ is inversely proportional to these basic flying qualities from Equation (8a) and (8b). Allowing a factor of two for static margin variations, the range of $d\delta e/d\delta s$ is:

Minimum $\dfrac{d\delta e}{d\delta s} = \pm 0.1$ deg/deg

Maximum $\dfrac{d\delta e}{d\delta s} = \pm 10$ deg/deg

Stick Servo

The pilot's control stick will also be actuated by an electronic-hydraulic servo system. Full stick travel of 31.5 deg. will be provided by 6 in. actuator travel. The signals to the stick servo are shown below:
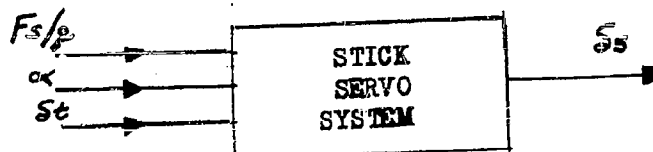
The stick force gradients will be varied by signals from F_s/q and α, while the δt signal will be used to trim the stick to zero force. The strain gages on the aluminum stick will measure strain equivalent to 10,000 psi stress when the stick force is 100 lbs. The dynamic pressure range for the F-94 will be from 46 to 340 psf.

The stick force per speed change (dF_s/dV_i) can be determined from Equation (7) as:

$$\frac{dF_s}{dV_i} \sim C_{h_\delta} \left[\frac{dC_m}{dC_L} - C_{m_{\delta e}} \frac{C_{h_{\alpha t}}}{C_{h_\delta}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \right]$$

The stick force per stick deflection can be expressed as a total derivative in terms of the servo gains as:

$$\left(\frac{dF_s}{d\delta_s} \right)_{Total} = \left(\frac{dF_s}{d\delta_s} \right)_{servo} + \left(\frac{dF_s}{d\delta_s} \right)_{servo} \times \left(\frac{d\delta_s}{d\alpha} \right)_{servo} \times \left(\frac{d\alpha}{d\delta_e} \right)_{servo} \times \left(\frac{d\delta_e}{d\delta_s} \right)_{airplane} \times \left(\frac{d\delta_e}{d\delta_s} \right)_{servo}$$

The servo gain $dF_s/d\delta_s$ is then proportional to the total derivative and is equivalent from Equation (11) to:

$$\frac{dF_s}{d\delta_s} \sim C_{h_{\delta_s}}$$

Variations in $C_{h_{\delta_s}}$ or $dF_s/d\delta_s$ will provide control over the flying quality dF_s/dV_i.

If a dF_s/dV_i range from slightly unstable to five times the normal stable value is required and a factor of two is allowed for $d\delta_e/d\delta_s$ or $d\delta_s/d\alpha$ variations, the range of $C_{h_{\delta_s}}$ and $dF_s/d\delta_s$ will be [8]

Maximum $C_{h_{\delta_s}} = 10\ C_{h_{\delta_e}}$ or $dF_s/d\delta_s = 107$ lbs/deg.
$\phantom{Maximum C_{h_{\delta_s}} = } -2\ C_{h_{\delta_e}}$ -21 lbs/deg.

A practical minimum range would be:

$C_{h_{\delta_s}} = \pm 0.1\ C_{h_{\delta_e}}$ or $dF_s/d\delta_s = \pm 1.1$ lbs/deg.

For a $C_{h_{\delta_s}}$ range of $\pm.2\ C_{h_{\delta_e}}$ to $^{+5}_{-2}\ C_{h_{\delta_e}}$ at the extreme limits of q, the normal

maximum $dF_s/d\delta_s$ will be

$dF_s/d\delta_s = 11$ lbs/deg at q = 46 psf

$dF_s/d\delta_s = 84$ lbs/deg at q = 340 psf

The signal $d\alpha/d\delta_s$ will be used to provide the dF_s/d_n gradients required. At $q = 218$ psf the range of stick force per g will be from -20 to +50 lbs/g. The minimum amount of control was calculated as a $d\alpha/d\delta_s$ equivalent to a $C_{h_{\alpha_t}}$ of

$\pm.10$ 1/rad. This is equivalent to:

$$\frac{d\delta's}{d\alpha} = - \frac{C_{h_{\alpha t}} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}{C_{h_{\delta s}}} = \pm \frac{(.10)\ (.475)}{(.482)\ 5} = \pm.020 \text{ deg/deg}$$

The stick force per g range can be provided if C_{h_α} be -1.05 and +1.30 1/rad. respectively for dF_s/d_n of -20 and +50 lbs/g. Including a factor of five as above for $C_{h_{\delta s}}$ variations, $d\delta_s/d\alpha$ will be:

Maximum negative $\quad \dfrac{d\delta s}{d\alpha} = - \dfrac{1.30\ (.475)}{.482\ \left(\frac{1}{5}\right)} = -6.4$ deg/deg

Maximum positive $\quad \dfrac{d\delta s}{d\alpha} = + \dfrac{1.05\ (.475)}{.482\ \left(\frac{1}{5}\right)} = +5.2$ deg/deg

Auxiliary Surface

The auxiliary surface will be actuated by a small electric servo motor geared down for a maximum 2 deg/sec surface rate. This velocity limit will allow the surface to control the phugoid without causing undesirable normal acceleration response to u gust inputs as shown later. This slow rate will be sufficient for phugoid control as a full range of +10 to -10 deg. surface travel will take 20 sec. or about one quarter of the phugoid period at 292 mph ind. The signals to the surface are shown below:
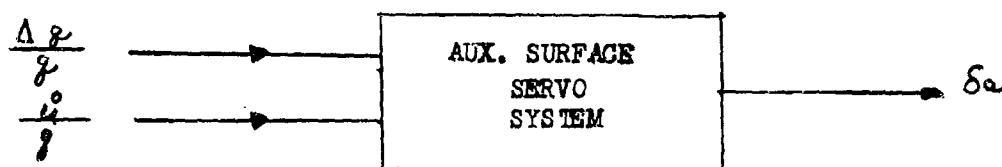
The auxiliary surface position signal proportional to $\Delta q/q$ will be provided to vary the phugoid period. The surface will have a ±10 degree position range. The largest control range was assumed at the $V_i = 292$ mph condition. A minimum Δq of 1 psf (or 1 mph) and a maximum Δq of ±29 psf were assumed. At the extreme q range, a Δq of ±5 psf at $q = 46$ psf and a Δq of ±24 psf at $q = 340$ psf were assumed.

The minimum $d\delta_a/d\left(\frac{\Delta q}{q}\right)$ signal will provide for 1% change in the equiv. static margin. Thus from Equations (5a) and (5b):

$$\frac{d\delta a}{d\frac{\Delta q}{q}} = \frac{C_{mu}}{2C_{m\delta a}} = \frac{2C_L}{2C_{m\delta a}} \left(\Delta \frac{dC_m}{dC_L}\right) = \pm \frac{.264}{.0002}\ (.01) = \pm13.2 \text{ deg.}$$

The maximum $d\delta_a/d\frac{\Delta \dot{z}}{\gamma}$ signal will allow a halving in the phugoid period at 5.5% static margin and V_i = 292 mph.

$$\frac{d\delta_a}{d\frac{\Delta \dot{z}}{\gamma}} = \frac{2C_L\left(-3\frac{dC_m}{dC_L}\right)}{2C_{m\delta_a}} = \frac{.264}{.0002}(+.165) = 218 \text{ deg.}$$

The maximum negative value is found if the period is to be doubled.

$$\frac{d\delta_a}{d\frac{\Delta \dot{z}}{\gamma}} = \frac{.264}{.0002}\left(\frac{3}{4}\right)(-.055) = -55 \text{ deg}$$

The signal proportional to $\dot{u}/q$ will be provided to change the phugoid damping a minimum of 5% critical at 5.5% static margin and V_i = 292 mph. From Equations (6a) and (6b)

$$\frac{d\delta_a}{d\frac{\dot{u}}{q}} = \frac{q\,\mathcal{T}}{V_i C_{m\delta_a}}\left\{4\Delta\zeta\left[2\frac{dC_m}{dC_L}\left(\frac{dC_m}{dC_L} + \frac{C_{m_q}}{2\mu}\right)\right]^{\frac{1}{2}}\right\}$$

$$= \frac{218\ (2.4)}{144\ (292)\ .0002}\left\{4(\pm.05)\left[2(-.055)(-.0661)\right]^{\frac{1}{2}}\right\} = \pm 1.06 \frac{\text{deg sec psi}}{\text{mph ind.}}$$

The maximum sensitivity of $d\delta_a/d\frac{\dot{u}}{q}$ will be required to increase the damping to 70% critical. Thus:

$$\frac{d\delta_a}{d\left(\frac{\dot{u}}{q}\right)} = \frac{218\ (2.4)}{144\ (292)\ .0002}\left\{4(.70 - .059)\left[2(-.055)(-.0661)\right]^{\frac{1}{2}}\right\} = 13.6 \frac{\text{deg sec psi}}{\text{mph ind}}$$

In order to reduce the damping to -20% critical:

$$\frac{d\delta_a}{d\left(\frac{\dot{u}}{q}\right)} = \frac{218\ (2.4)}{144\ (292)\ .0002}\left\{4(-.259)\left[2(-.055)(-.0661)\right]^{\frac{1}{2}}\right\} = -5.7 \frac{\text{deg sec psi}}{\text{mph ind.}}$$

Summary

Derivative

Elevator Servo

$d\delta_e/d\alpha$ $\pm\ .0625$ to ${}^{+\ 2.06}_{-\ .51}$ deg/deg.

$d\delta_e/d\dot{\alpha}$ $\pm\ .025$ to ${}^{+\ .48}_{-\ .23}$ deg/deg/sec.

$d\delta_e/d\delta_s$ $\pm\ 0.1$ to ${}^{+}_{-}\ 10$ deg/deg.

Stick Servo

$dF_s/d\delta_s$ ${}^{+}_{-}\ 1.1$ to ${}^{+\ 107}_{-\ 21}$ lbs/deg.

$d\delta_s/d\alpha$ ${}^{+}_{-}\ .02$ to ${}^{+\ 5.2}_{-\ 6.4}$ deg/deg.

Auxiliary Surface Servo

$d\delta_a/d\,\dfrac{\Delta q}{q}$ ${}^{+}_{-}\ 13.2$ to ${}^{+\ 218}_{-\ 55}$ deg.

$d\delta_a/d\,\dfrac{\dot{u}}{q}$ ${}^{+}_{-}\ 1.06$ to ${}^{+\ 13.6}_{-\ 5.7}\left(\dfrac{\text{deg/sec/psi}}{\text{mph ind.}}\right)$

TRANSIENT RESPONSE

The response of the F-94 at 292 mph indicated airspeed and 20,000 ft. altitude was determined by analog computation for three disturbance conditions. A step C_m was introduced on the analog by either a sudden application of 1 deg. elevator or 10 deg. auxiliary surface deflection. Gusts in u and α were entered as initial conditions. A description of the computational approach can be found in the appendix.

Pitching Moment Steps

Figure (1) shows the F-94 response to a 1 deg. step elevator deflection for the normal condition of V_i = 292 mph, 20,000 ft. altitude, 5.5% static margin and gross weight of 13,614 lbs. The heavily damped short period is evident in the slight angle of attack overshoot. The low phugoid damping is also apparent.

The effects of ΔC_{m_α} and $\Delta C_{m_{D\alpha}}$ are seen in Figure (2). The damping of the short period is increased with negative $\Delta C_{m_{D\alpha}}$. The elevator trace shows that about 0.2 deg of elevator control would be needed with this artificial control for 1 deg step elevator input. If negative ΔC_{m_α} is added the spring constant of the motion changes with a corresponding increase in frequency. The 1 deg. elevator step now creates less α and n_z response due to the increased stability. Over 0.7 deg. elevator control is required with this artificial derivative.

Figure (3) reveals the phugoid mode variations with C_{m_u} and $C_{m_{D u}}$. Positive $C_{m_{D u}}$ of .171 adds 50% critical damping and requires almost 5 deg. of auxiliary surface control for each 10 deg. of excitation. The decreased period due to C_{m_u} = .029 is evident in all the traces. This artificial control demands almost as wide a variation in δ_a as the excitation itself as seen in the auxiliary surface trace.

Forward Velocity Gust

The response of the normal F-94 to a 10 mph ind. forward velocity gust is seen in Figure (4). This gust as represented by an initial condition on u causes an initial value of n at zero time. After the short period α peak, the motion is characterized by the lightly damped phugoid mode. If .029 C_{m_u} is added by artificial control the gust response changes to that shown in Figure (5). The g response increases somewhat after its initial value. Approximately 7 deg. of auxiliary surface deflection would be required to provide this artificial C_{m_u} in response to a 10 mph gust.

Figure (6) shows the effect of $C_{m_{D_{w}}}$ on the normal acceleration response due to a forward velocity gust. In the normal case at the left, the in itial g decreases steadily after the short period dip. If a $C_{m_{D_{w}}}$ of .171 is added to the airplane, the response shows a sharp peak about seven times the initial normal acceleration amplitude. This effect is due to the impulse in C_m created by use of $C_{m_{D_{w}}}$, as explained in Reference (1). The use of a velocity limiter on the right hand graph of Figure (6) reveals the close to normal response obtained by limiting the rate of buildup of δ_{a} to 2 deg/sec.

Angle of Attack Gust

The effects of $\triangle C_{m_{\alpha}}$ and $\triangle C_{m_{D_{\alpha}}}$ on the F-94 response to an angle of attack gust is seen in Figure (7). The magnitude of the gust input varies from .352 deg. for the normal airplane and for $\triangle C_{m_{\alpha}}$, to 1 deg. for the $\triangle C_{m_{D_{\alpha}}}$ condition. This limitation arose from computer overload problems. Overshoot in angle of attack, pitch rate and normal acceleration is evident in all but the $\triangle C_{m_{D_{\alpha}}}$ condition. When 80% critical damping of the short period is caused with $\triangle C_{m_{D_{\alpha}}}$ the aircraft responds smoothly to return to the equilibrium condition. The use of $\triangle C_{m_{D_{\alpha}}}$ = -.043 would require about 0.2 deg. elevator per deg. angle of attack gust, while use of $\triangle C_{m_{\alpha}}$ = -.389 requires 0.6 deg. δ_{e} per deg. α gust. The increase in the short period frequency by $\triangle C_{m_{\alpha}}$ is evident in the faster initial response to the gust input..

It should be noted that the transient curves presented were transcribed directly from Brush recorder analog results. Thus, the time scale is correct only at the equilibrium value. A similar time at any amplitude can be found on a circular arc of 3 inch radius. It was not deemed important to alter this time scale to the usual rectangular coordinates.

VECTOR PHASE DIAGRAMS

The homogeneous equations of motion of the aircraft can be solved for the vector balance of forces and inertias required to maintain equilibrium at any frequency of oscillation. This vector balance is illustrated in Reference (3). The method of obtaining the phasings of the motions in the phugoid and in the short period is presented in the Appendix.

Short Period

The short period phasing for the F-94A airplane at $V_1 = 292$ mph, 20,000 ft. altitude and 5.5% static margin is presented in Figure (8). It can be seen that for the normal airplane the $D\theta$ response will be 4.7 times the α response and 87.2° ahead of α. The amplitude and phase of $D\alpha$ in relation to α indicates the .314 cps frequency and 49.5% critical damping of the motion. The unimportance of u in the short period is apparent.

The effects of ΔC_{m_α} and $C_{m_{D^2\theta}}$ on the motions are illustrated in Figure (8). Both artificial derivatives raise the frequency to .475 cps, as can be noted in the similar amplitude of the $D\alpha$ vectors. The use of ΔC_{m_α} will lower the damping, however, as seen by the smaller lead angle between $D\alpha$ and α. The $D\theta$ response due to ΔC_{m_α} will be almost in phase with the normal $D\theta$ vector, while $C_{m_{D^2\theta}}$ will cause $D\theta$ to lead α by 101.3°.

Figure (9) shows the effects of $\Delta C_{m_{D\alpha}}$ and ΔC_{m_q} on the responses. Both derivatives increase the damping to 79.5% critical, but ΔC_{m_q} also causes an increase in frequency as seen by the larger amplitude of the $D\alpha$ vector. The increase in $D\theta$ lead over α is apparent for both artificial derivatives, with $\Delta C_{m_{D\alpha}}$ also decreasing the amplitude of the $D\theta$ vector considerably.

Phugoid

The phugoid phasing for the F-94A condition is shown in Figure (10). The low damping (5.9% critical) is apparent in the Du lead of u by only 93.4°. The high period of 88.6 sec. is noted in the length of the Du vector. The phugoid motion occurs at almost constant angle of attack, as noted by the minute α vector, while the pitch angle variation is quite important being 1.28 times the amplitude of u or $\left(\dfrac{\Delta V}{V}\right)$ and lagging u by 95°.

Figure (10) also illustrates the effects of C_{m_u} and $C_{\not{\downarrow}_{Du}}$ on the phugoid motions. While each artificial derivative reduces the period to 62.8 sec. C_{m_u} decreases ζ_p somewhat, and $C_{\not{\downarrow}_{Du}}$ causes a slight increase over the normal damping. The pitch angle response per unit is raised to 1.81 by C_{m_u} and lowered to 0.90 by $C_{\not{\downarrow}_{Du}}$.

The phugoid damping can be increased to 56.1% critical by addition of $C_{m_{Du}}$, $C_{\not{\downarrow}_u}$, or C_{m_Θ} as seen in Figure (11). All three artificial derivatives maintain the period close to the normal 88.6 sec. value. A significant increase in angle of attack response is noted with the use of C_{m_Θ} or $C_{m_{Du}}$. The pitch angle lag is reduced to 54° with $C_{m_{Du}}$ and 52° with C_{m_Θ} while $C_{\not{\downarrow}_u}$ increases the lag to 128°.

The usefulness of these phase diagrams is apparent from the ease with which the amplitude and phasings of the motions can be obtained. It should be noted that these diagrams can be obtained quite readily by graphical analysis. Besides enabling the theoretical analyst to gain a physical insight into the motions, these diagrams can be used to predict the transient response of other variables, once the response of a single variable has been determined. Thus, assume that the $D\Theta$ response to a particular input has been determined as:

$$D\Theta = C_1 e^{-\alpha_1 t} \cos(\beta_1 t + \psi_1) + C_2 e^{-\alpha_2 t} \cos(\beta_2 t + \psi_2) + D\Theta_{\text{part.}}$$

where $D\Theta$ part. = particular solution for $D\Theta$. The response to any other variable X can be determined as:

$$X = A_{X_1} C_1 e^{-\alpha_1 t} \cos(\beta_1 t + \psi_1 + \phi_1) + A_{X_2} C_2 e^{-\alpha_2 t} \cos(\beta_2 t + \psi_2 + \phi_2)$$

$$+ X_{\text{part.}}$$

where $A_{\dot{x}1}$ = amplitude ratio of X to $D\theta$ in short period (β_1 = short period frequency).

A_{x2} = amplitude ratio of X to $D\theta$ in phugoid (β_2 = phugoid frequency)

ϕ_1 = phase lead of X from $D\theta$ in short period

ϕ_2 = phase lead of X from $D\theta$ in phugoid

$X_{part.}$ = particular solution for X

The new particular response to the input disturbance must be calculated separately. In the case of step disturbances or initial conditions this particular response can be found for combined phugoid and short period analysis from the formulae:

Step Elevator Particular Solutions

$$u_{part.} = u_{ss}$$

$$\theta_{part.} = \theta_{ss}$$

$$\alpha_{part.} = \alpha_{ss}$$

$$n_{z\ part.} = 0$$

See formulae for steady state values on Page 43 in Appendix

Particular Solutions When Input is Initial Condition on u or α

$$u_{part.} = \theta_{part.} = \alpha_{part.} = n_{z\ part.} = 0$$

With the particular solutions known, the response in the desired motion, X, can be readily determined with the amplitude and phase ratios (A_x and ϕ_x) determined at both short period and phugoid frequencies.

RESULTS AND CONCLUSIONS

On the basis of this theoretical investigation to provide a wide range of artificial stability and control to the F-94 airplane, the following results and conclusions can be noted:

1. Use of elevator signals proportional to α and $\overset{c}{\alpha}$ will provide a wide range of short period frequency and damping control.

2. Use of auxiliary surface position signals proportional to $\Delta q/q$ and u/q will provide a wide range of phugoid period and damping control.

3. Use of elevator position signals varying with stick position will change the fixed stick stability.

4. Use of stick force signals varying with stick position and angle of attack will change the free stick stability of the airplane.

5. Seven artificial stability derivatives will be provided on the F-94 with a hydraulic servo system on the elevator and stick, and with an electric servo motor on the auxiliary surface.

6. An auxiliary surface of 0.43 sq. ft. will be constructed and installed in the nose section for phugoid control. The surface will be geared down to 2 deg/sec. to avoid normal acceleration peaks due to forward velocity gusts with $C_{m_{ou}}$ present.

7. The phasings of the phugoid and short period motions can be investigated readily by vector phase diagrams.

8. The phasings of the oscillatory modes may be an important parameter in the pilot's evaluation of a flight configuration. Further insight into the effect on the human pilot of the phasings in the short period and phugoid response should be gained by analysis of flight traces as well as by theoretical studies.

BIBLIOGRAPHICAL REFERENCES

1. Heilenday, F. W.   Artificial Stability and Control of Longitudinal
   Motion of the B-26 Aircraft - Theoretical Investigation.   AF Technical
   Report No. 6703 (also Cornell Aeronautical Laboratory Report No.
   TB-757-F-2)   November 1951.

2. Campbell, G. F.   Artificial Stability and Control in Military
   Aircraft - Quarterly Progress Report.   Cornell Aeronautical
   Laboratory Report No. TB-757-F-3.   17 January 1952.

3. Mueller, R. K.   The Graphical Solution of Stability Problems.
   Journal of the Aeronautical Sciences Vol. 4 No. 8   June 1937.

TABLE I

F-94A NORMAL STABILITY DERIVATIVES

Constants:

$c = 6.72$ ft.

$S = 237.6$ ft.^{2}

$\dfrac{d\epsilon}{d\alpha} = .525$

$S_t = 47.83$ ft.^{2}

$\dfrac{q_H}{q} = 0.90$

$c_e = .720$ ft.

$\dfrac{\partial F_s}{\partial H} = 1.03$ rad./ft.

$S_e = 8.70$ ft.^{2}

Variables:

Indicated Airspeed, V_i (mph)		135	219	292	·290	365
Mach Number		.18	.43	.57	.70	.71
True Airspeed, V (mph)		135	300	400	474	500
Altitude (ft.)		S.L.	20,000	20,000	30,000	20,000
C.G. (%MAC)		28	29	29	29	29
dC_m/dC_L		-.049	-.055	-.055	-.055	-.055
G.W. (lbs.)		12,359	13,614	13,614	11,365	13,614
I_y (slug ft.^{2})		26,545	26,543	26,543	22,158	26,543
i_B		6.12	5.54	5.54	5.54	5.54
$2\,l_t/c$		5.23	5.21	5.21	5.21	5.21
δ_f (deg.)		45	0	0	0	0
C_L		1.12	.466	.264	.222	.168
C_{L_α} (1/rad)		5.12	5.51	5.90	6.48	6.41
C_D		.135	.0307	.0235	.0242	.0237
C_{D_α} (1/rad)		.653	.264	.147	.140	.110
C_{m_α} (1/rad)		-.251	-.304	-.324	-.357	-.353
$C_{m_{\delta_e}}$ (1/deg)		-.0155	-.0165	-.0165	-.0165	-.0165
C_{m_q} (1/rad)		-8.96	-9.28	-9.28	-9.28	-9.28
$C_{m_{D\alpha}}$ (1/rad)		-4.10	-4.24	-4.24	-4.24	-4.24
τ (sec.)		3.45	3.20	2.40	2.41	1.92
μ		203	419	419	497	419
$C_{h_{\delta_e}}$ (1/rad)		-.430	-.478	-.482	-.460	-.455
$C_{h_{\alpha_t}}$ (1/rad)		-.15	-.10	-.09	-.09	-.06

TABLE II

CHARACTERISTIC ROOTS OF THE NORMAL F-94A

$\dfrac{dC_m}{dC_L}$	V_i (mph)	Phugoid		Short Period	
		Period (sec.)	% Critical Damping	Frequency (cps)	% Critical Damping
-.049	135	33.1	8.35	.119	67.4
-.055	219	67.0	3.25	.227	49.5
-.01	292	118	7.61	.0991	87.5
-.055	292	88.6	5.94	.314	49.5
-.10	292	85.7	5.98	.431	38.2
-.055	290	104.2	7.53	.361	46.1
-.055	365	112	10.3	.408	49.9

TABLE III

POSSIBLE ARTIFICIAL CONTROL OF THE SHORT PERIOD

Parameter and Nondimensional coefficient	Primary Short Period Effect	Amount of control required for F-94 at V_1 = 292 mph C.G. at 29% MAC H = 20,000 ft. GW = 13,614 lbs.	Remarks
$\dfrac{\partial \text{ Pitching Moment}}{\partial \text{ Angle of Attack}} = \Delta C_{m_\alpha}$	Varies frequency	$\Delta C_{m_\alpha} = -.339$ to increase frequency from .314 to .475 cps $\dfrac{\partial \delta_e}{\partial \alpha} = .412$ deg/deg	Phugoid damping slightly increased. Phugoid period decreased from 88.6 to 84.9 sec. Short period damping reduced from 49.5 to 35.1% critical. Affects static stability.
$\dfrac{\partial}{\partial \dfrac{d}{dt}} \dfrac{\text{Pitching Moment}}{\text{(Angle of Attack)}} = \Delta C_{m_{\dot\alpha}}$	Varies damping	$\Delta C_{m_{\dot\alpha}} = -.043$ to add 30% critical damping $\dfrac{\partial \delta_e}{\partial \dot\alpha} = .109 \dfrac{\text{deg.sec.}}{\text{deg.}}$	Phugoid roots are not changed. Frequency of short period decreased from .314 to .219 cps.
$\dfrac{\partial \text{ Pitching Moment}}{\partial \text{ Pitch Rate}} = \Delta C_{m_q}$	Varies damping	$\Delta C_{m_q} = -29.1$ to add 30% critical damping $\dfrac{\partial \delta_e}{\partial \dot\theta} = .176 \dfrac{\text{deg.sec.}}{\text{deg.}}$	Phugoid damping and period increased from 5.9 to 8.0% critical and from 88.6 to 110 sec. respectively. Short period frequency decreased from .314 to .270 cps. Affects maneuver stability.
$\dfrac{\partial \text{ Pitching Moment}}{\partial \dfrac{d^2}{dt^2} \text{ (Pitch Angle)}} = C_{m_{\ddot\theta}}$	Varies frequency	$C_{m_{\ddot\theta}} = .0077$ to increase frequency from .314 to .475 cps $\dfrac{\partial \delta_e}{\partial \ddot\theta} = -.047 \dfrac{\text{deg.sec.}^2}{\text{deg.}}$	Phugoid roots are not changed. Short period damping increased slightly from 49.5 to 52.5% critical. The characteristics of the short period will be altered if $C_{m_{\ddot\theta}}$ is increased (or the inertia decreased) beyond this value.

TABLE IV

POSSIBLE ARTIFICIAL CONTROL OF THE PHUGOID

| Parameter and Nondimensional coefficient	Primary Phugoid Effect	Amount of control required for F-94 at V_i = 292 mph C.G. at 29% MAC H = 20,000 ft. G.W. = 13,614 lbs.	Remarks
$\dfrac{\partial \text{Thrust}}{\partial \text{Airspeed}} = C_{xu}$	Varies damping	C_{xu} = -.17 to add 50% critical damping. $\dfrac{\partial T}{\partial V_i}$ = -61 lbs/mph ind.	Short period roots are not changed. Phugoid period increased from 88.6 to 107 sec.
$\dfrac{\partial \text{Thrust}}{\partial(\frac{d}{dt}\text{Airspeed})} = C_{xou}$	Varies period	C_{xou} = .501 to reduce period to 62.8 sec. $\dfrac{\partial T}{\partial \dot{V}_i}$ = 426 $\dfrac{\text{lbs.sec.}}{\text{mph ind.}}$	Short period roots are not changed. Phugoid damping increased from 5.9 to 8.4% critical
$\dfrac{\partial \text{Thrust}}{\partial \text{Pitch Angle}} = C_{x\theta}$	Varies period	$C_{x\theta}$ = -.132 to reduce period to 62.8 sec. $\dfrac{\partial T}{\partial \theta}$ = -238 lbs/deg.	Short period roots are not changed. Phugoid damping decreased from 5.9 to 3.5% critical
$\dfrac{\partial \text{Thrust}}{\partial \text{Pitch Rate}} = C_{x\theta\theta}$	Varies damping	$C_{x\theta\theta}$ = -.779 to add 50% critical damping. $\dfrac{\partial T}{\partial \dot{\theta}}$ = -3380 $\dfrac{\text{lbs.sec.}}{\text{deg.}}$	Short period roots are not changed. Phugoid period increased from 88.6 to 107 sec.
$\dfrac{\partial \text{Pitching Moment}}{\partial \text{Airspeed}} = C_{mu}$	Varies period	C_{mu} = .029 to reduce period to 62.8 sec. $\dfrac{\partial \delta e}{\partial V_i}$ = -.0059 $\dfrac{\text{deg}}{\text{mph ind.}}$	Short period roots are not changed. Phugoid damping decreased from 5.9 to 4.2% critical
$\dfrac{\partial \text{Pitching Moment}}{\partial(\frac{d}{dt}\text{Airspeed})} = C_{mou}$	Varies damping	C_{mou} = .171 to add 50% critical damping. $\dfrac{\partial \delta e}{\partial \dot{V}_i}$ = -.085 $\dfrac{\text{deg.sec.}}{\text{mph ind.}}$	Short period roots are not changed. Phugoid period increased from 88.6 to 107 sec.
$\dfrac{\partial \text{Pitching Moment}}{\partial(d^2/dt\,\text{Airspeed})} = C_{mo^2u}$	Varies period	C_{mo^2u} = -.503 to reduce period to 62.8 sec. $\dfrac{\partial \delta e}{\partial \dot{V}_i}$ = .60 $\dfrac{\text{deg sec.2}}{\text{mph ind.}}$	Damping and frequency of short period are decreased. Phugoid damping increased from 5.9 to 8.5 % critical
$\dfrac{\partial \text{Pitching Moment}}{\partial \text{Pitch Angle}} = C_{m\theta}$	Varies damping	$C_{m\theta}$ = -.025 to add 50% critical damping. $\dfrac{\partial \delta e}{\partial \theta}$ = .26 deg/deg	Short period roots are not changed. Phugoid period increased from 8?? to ?? 5 s...

TABLE V

CONTROL STICK PARAMETERS OF THE NORMAL F-94A

$\dfrac{dC_m}{dC_L}$	V_i (mph)	$d\delta_e/dn$ (deg/g)		dF_s/dn (lbs/g)		$d\delta_e/dV_i$ $V_i = V_{trim}$ (deg/mph ind)	dF_s/dV_i $V_i = V_{trim}$ (lbs/mph ind)
		Push Downs	Turns	Push Downs	Turns		
-.049	135	4.36	$4.36 + .822/n^2$	4.35	$4.36 + 1.39/n^2$	.0524	.0438
-.055	219	1.879	$1.879 + .314/n^2$	8.06	$8.06 + 1.66/n^2$	.0144	.0585
-.01	292	.338	$.338 + .177/n^2$	.977	$0.977 + 1.69/n^2$	.00109	-.00488
-.055	292	1.059	$1.059 + .177/n^2$	8.60	$8.60 + 1.69/n^2$	.00601	.0474
-.10	292	1.782	$1.782 + .177/n^2$	16.24	$16.24 + 1.69/n^2$	.0109	.0996
-.055	290	.923	$.923 + .179/n^2$	7.21	$7.21 + 1.61/n^2$	.00512	.0386
-.055	365	.677	$.677 + .113/n^2$	9.01	$9.01 + 1.65/n^2$	.00309	.0403

FIGURE 1

F - 94
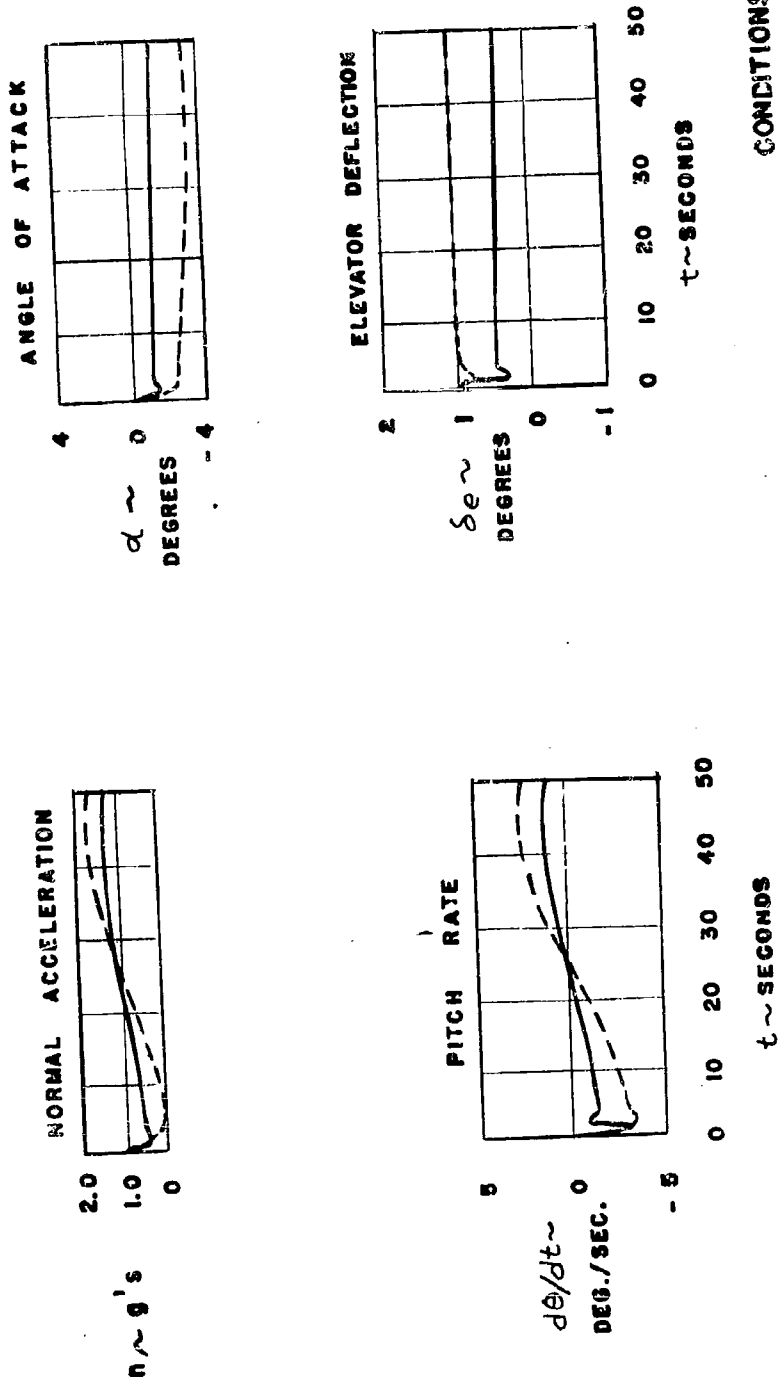# TRANSIENT RESPONSE TO STEP ELEVATOR
NORMAL AIRPLANE
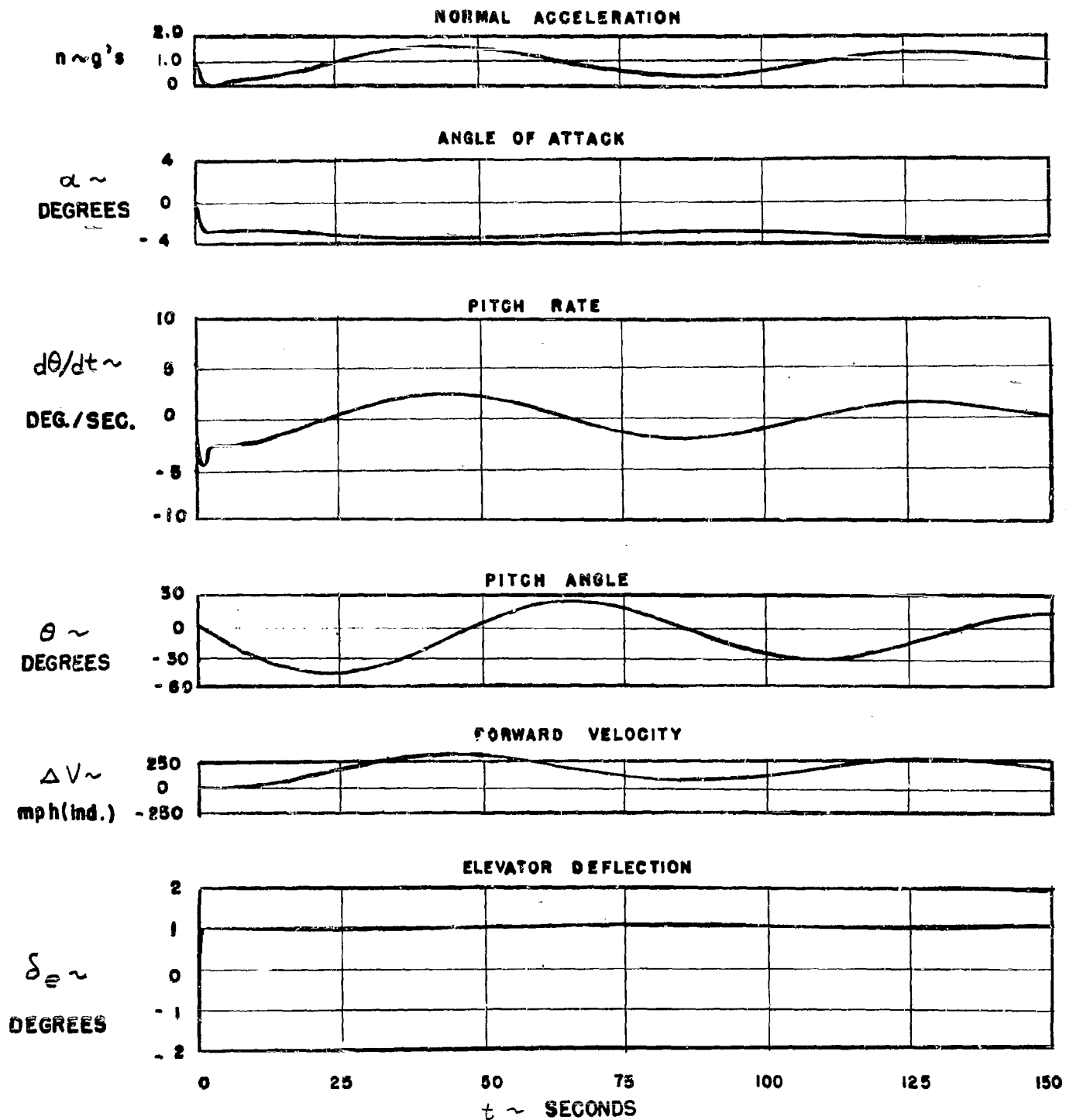
CONDITIONS

$V_i = 292$ mph
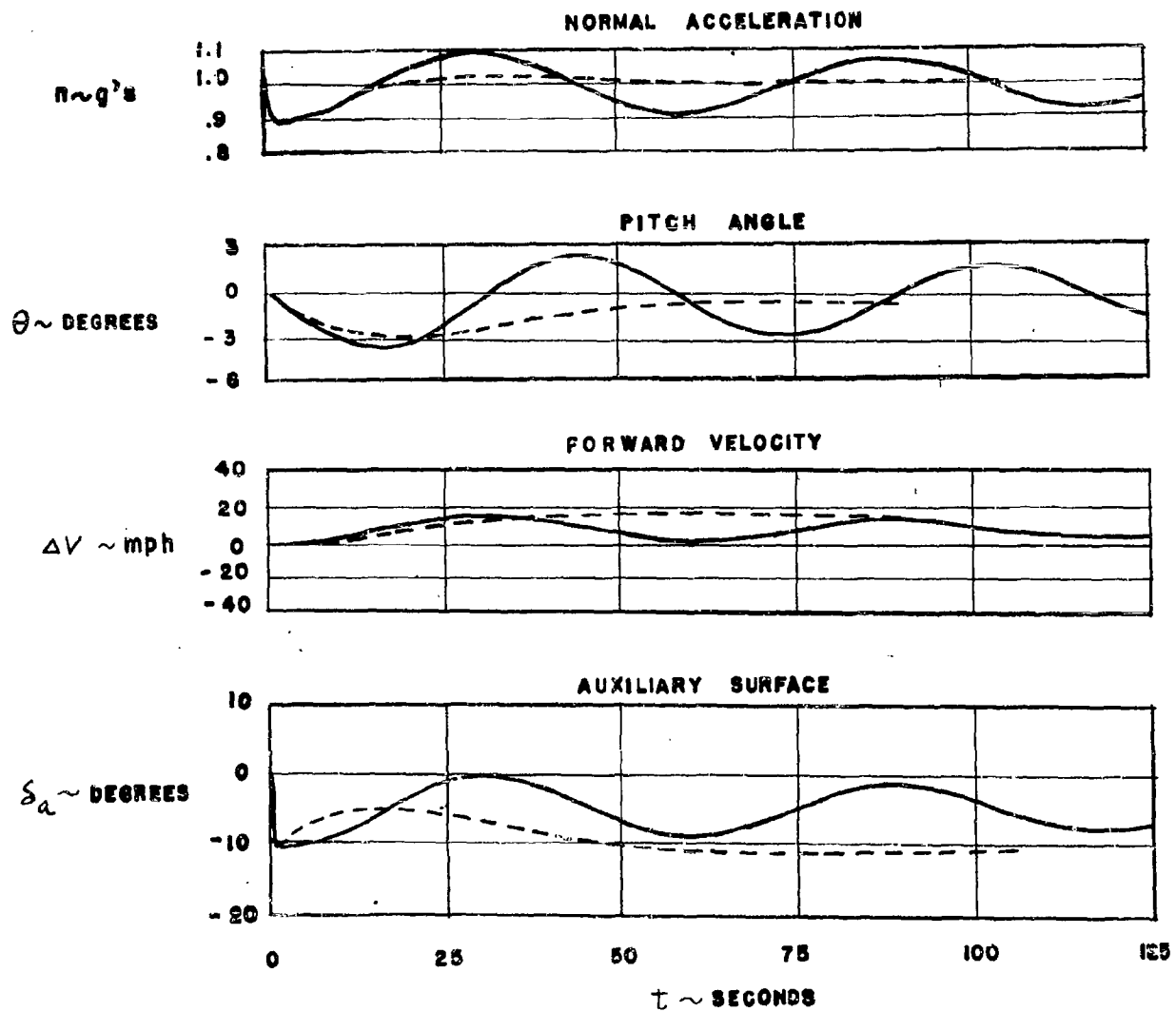$H = 20,000$ ft.
$dC_m / dC_L = -.055$
G.W. = 13,614 lbs.

FIGURE 2

TRANSIENT RESPONSE TO STEP ELEVATOR
F-94
SHORT PERIOD EFFECTS, $\Delta C_{m_\alpha} = -.389$, $\Delta C_{m_{D\alpha}} = -.043$

ANGLE OF ATTACK

$\alpha \sim$ DEGREES

ELEVATOR DEFLECTION

$\delta e \sim$ DEGREES

$t \sim$ SECONDS

NORMAL ACCELERATION

$n \sim g$'s

PITCH RATE

$d\theta/dt \sim$ DEG./SEC.

$t \sim$ SECONDS

$\Delta C_{m_\alpha} = .389$

$\Delta C_{m_{D\alpha}} = -.043$

CONDITIONS

$V_i = 292$ mph
$H = 20,000$ ft.
$dC_m/dC_L = -.055$
G. W. = 13,614 lbs.

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TRANSIENT RESPONSE TO STEP AUXILIARY SURFACE DEFLECTION

PHUGOID EFFECTS, $C_{m_u} = .029$, $C_{m_{Du}} = .171$

NORMAL ACCELERATION

$n \sim g's$

PITCH ANGLE

$\theta \sim$ DEGREES

FORWARD VELOCITY

$\Delta V \sim mph$

AUXILIARY SURFACE

$\delta_a \sim$ DEGREES

$t \sim$ SECONDS

———— $C_{m_u} = .029$

– – – – $C_{m_{Du}} = .171$

CONDITIONS

$V_i = 292$ mph
$H = 20,000$ ft.
$dC_m / dC_L = -.035$
G.W. $= 13,614$ lbs.

TRANSIENT RESPONSE TO FORWARD VELOCITY GUST
NORMAL AIRPLANE

NORMAL ACCELERATION

$n \sim g$'s

ANGLE OF ATTACK

$\alpha \sim$ DEGREES

PITCH RATE

$d\theta/dt \sim$ DEG. / SEC.

PITCH ANGLE

$\theta \sim$ DEGREES

FORWARD VELOCITY

$\Delta V \sim$ mph ind.

$t \sim$ SECONDS
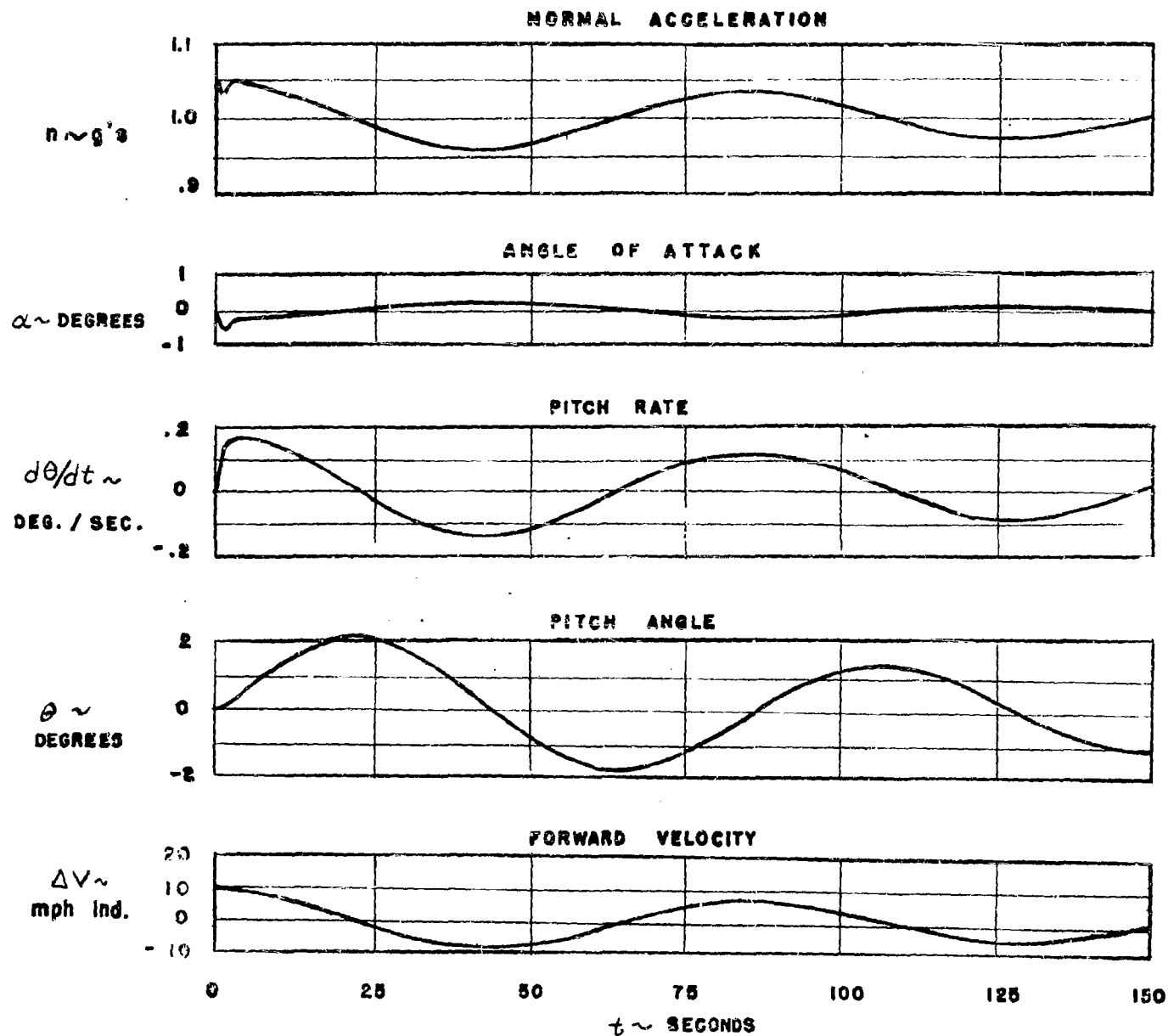
CONDITIONS

$V_i = 292$ mph

$H = 20,000$ ft.

$dC_m/dC_L = -.055$

G. W. $= 13,614$ lbs.

FIGURE 5

TRANSIENT RESPONSE TO FORWARD VELOCITY GUST

$$C_{m_u} = .029$$

NORMAL ACCELERATION

ANGLE OF ATTACK

PITCH RATE

PITCH ANGLE

AUXILIARY SURFACE

FORWARD VELOCITY

$t \sim$ SECONDS
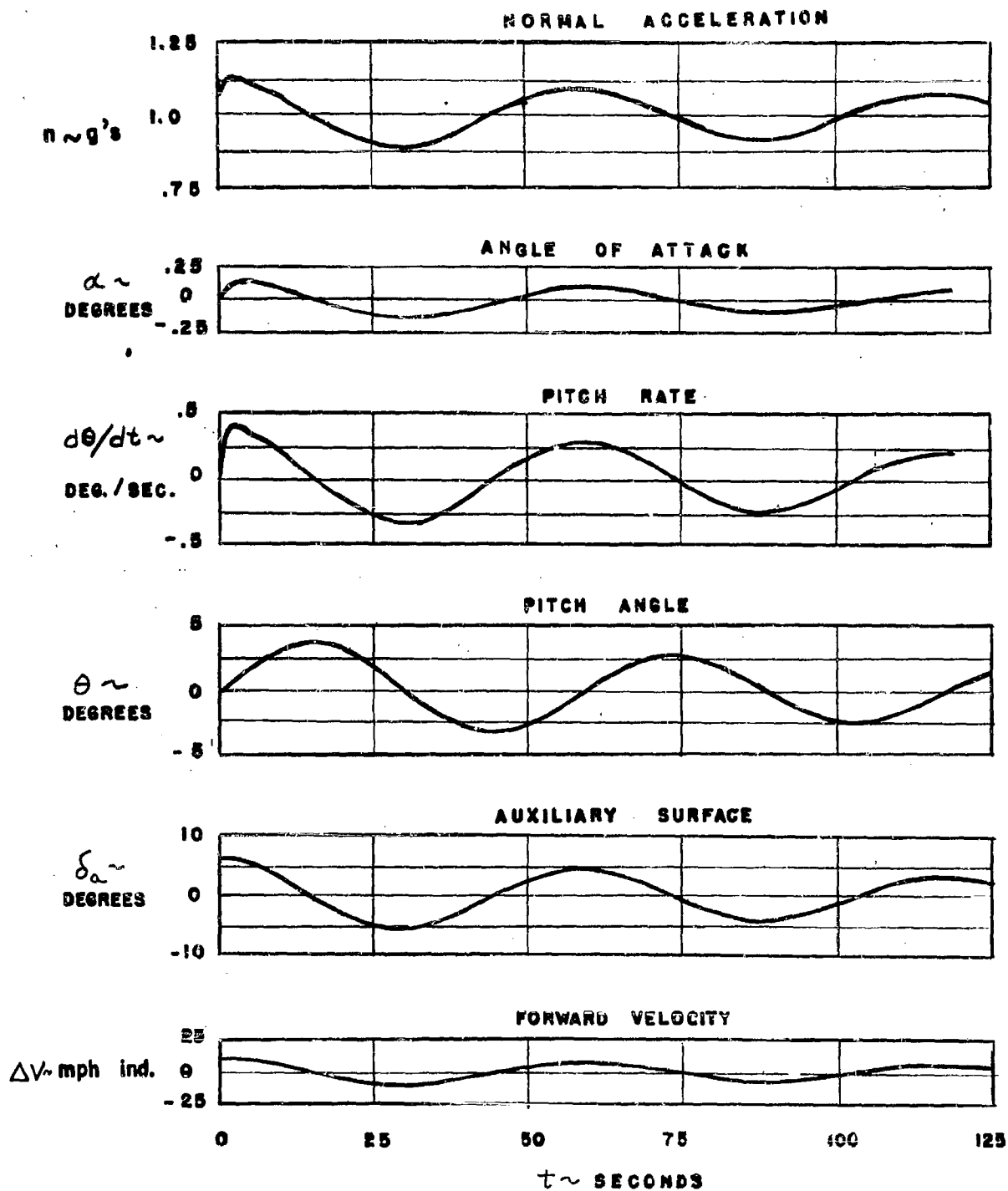
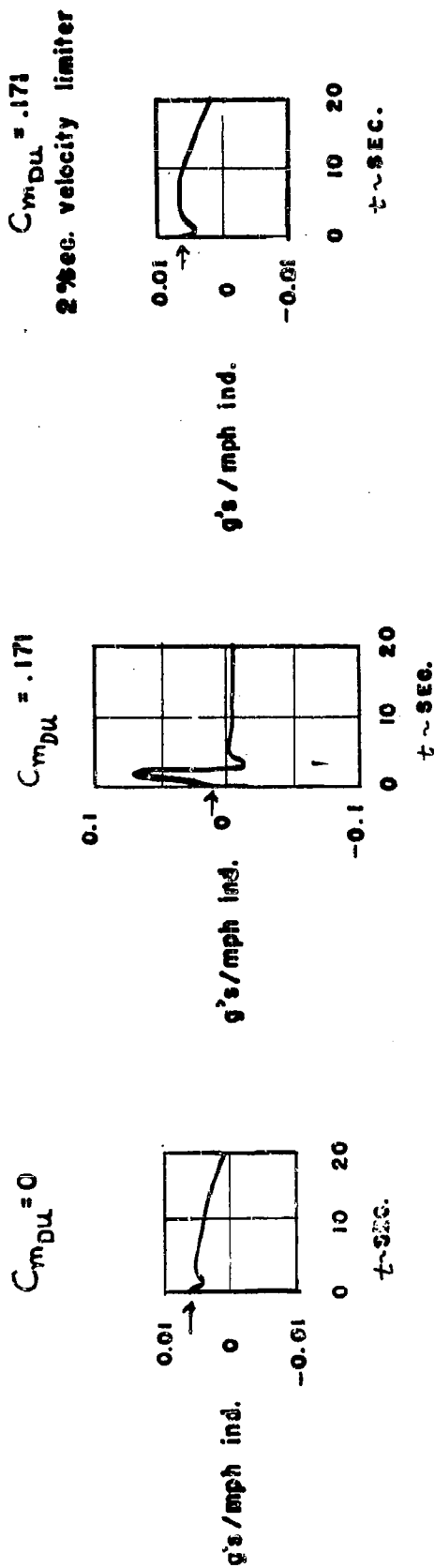**CONDITIONS**

V_i = 292 mph
H = 20,000 ft.
dC_m/dC_L = -.058
G. W. = 13,614 lbs.

FIGURE 6

F-94

TRANSIENT RESPONSE TO FORWARD VELOCITY GUST

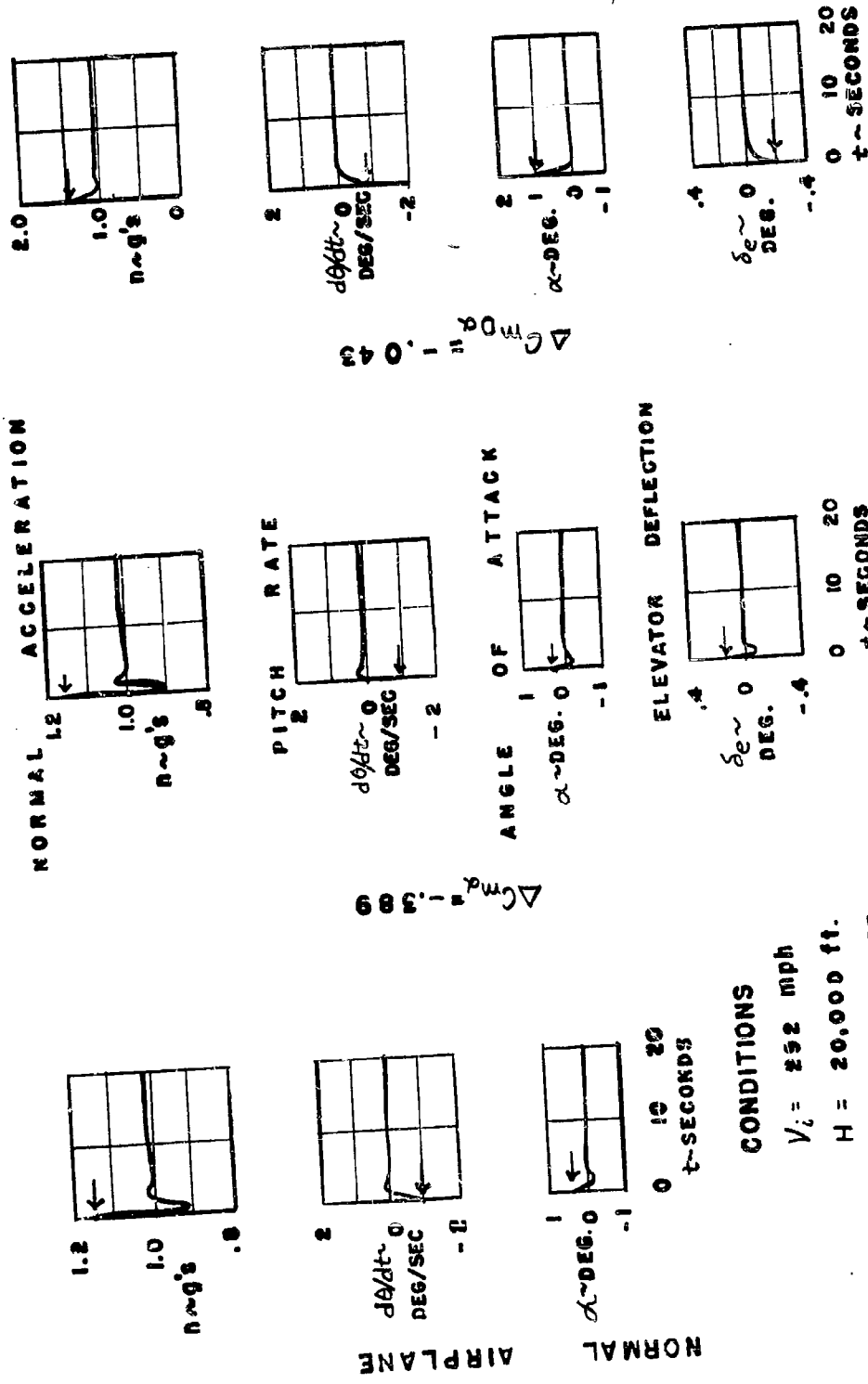## EFFECT OF $C_{m_{DU}}$ ON NORMAL ACCELERATION

$C_{m_{DU}} = 0$

$C_{m_{DU}} = .171$

$C_{m_{DU}} = .171$
2 sec. velocity limiter

$\longleftarrow$ DENOTES VALUE AT ZERO TIME

CONDITIONS

$V_i = 292$ mph
$H = 20,000$ ft.
$dC_m/dC_L = -.055$
G. W. $= 13,814$ lbs.

FIGURE 7

TRANSIENT RESPONSE TO ANGLE OF ATTACK GUST

F-94

NORMAL ACCELERATION

$n \sim g's$ — 1.2, 1.0, .8

PITCH RATE

$d\theta/dt \sim$ DEG/SEC — 2, 0, -2

ANGLE OF ATTACK

$\alpha \sim$ DEG. — 1, 0, -1

ELEVATOR DEFLECTION

$\delta_e \sim$ DEG. — .4, 0, -.4

$t \sim$ SECONDS — 0, 10, 20

$\Delta C_{m_\alpha} = -.389$

$n \sim g's$ — 2.0, 1.0, 0

$d\theta/dt \sim$ DEG/SEC — 2, 0, -2

$\alpha \sim$ DEG. — 2, 1, 0, -1

$\delta_e \sim$ DEG. — .4, 0, -.4

$t \sim$ SECONDS — 0, 10, 20

$\Delta C_{m_\alpha} = -.043$

$n \sim g's$ — 1.2, 1.0, .8

$d\theta/dt \sim$ DEG/SEC — 2, 0, -2

$\alpha \sim$ DEG. — 1, 0, -1

$t \sim$ SECONDS — 0, 10, 20

AIRPLANE NORMAL

CONDITIONS

$V_i = 252$ mph

$H = 20,000$ ft.

$dC_m/dC_L = -.055$

G. W. = 13,614 lbs.

$\longrightarrow$ DENOTES VALUE AT ZERO TIME

FIGURE 8

F - 94
SHORT PERIOD PHASING
FREQUENCY VARIED
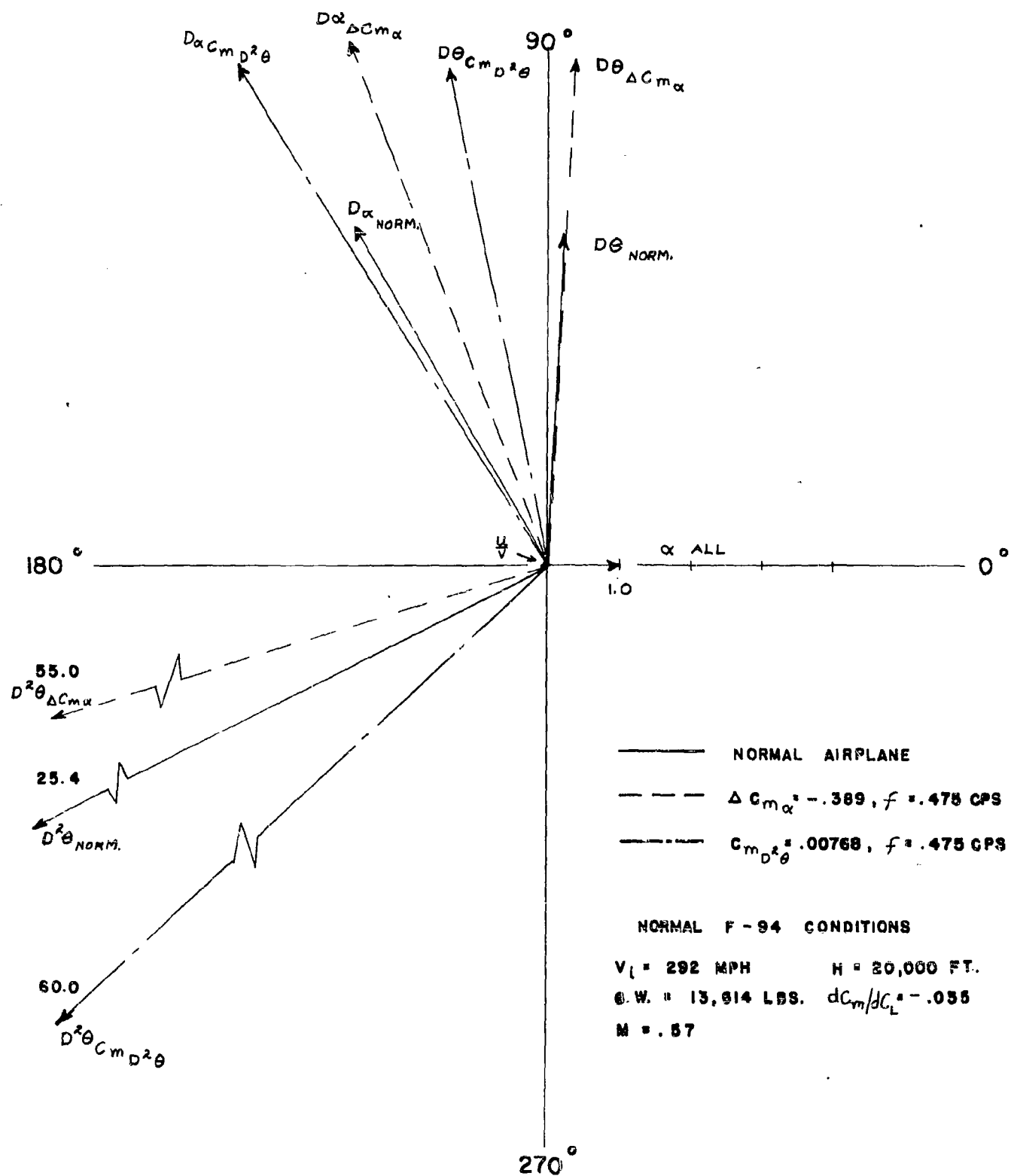
FIGURE 9

F - 94

SHORT PERIOD PHASING

DAMPING VARIED

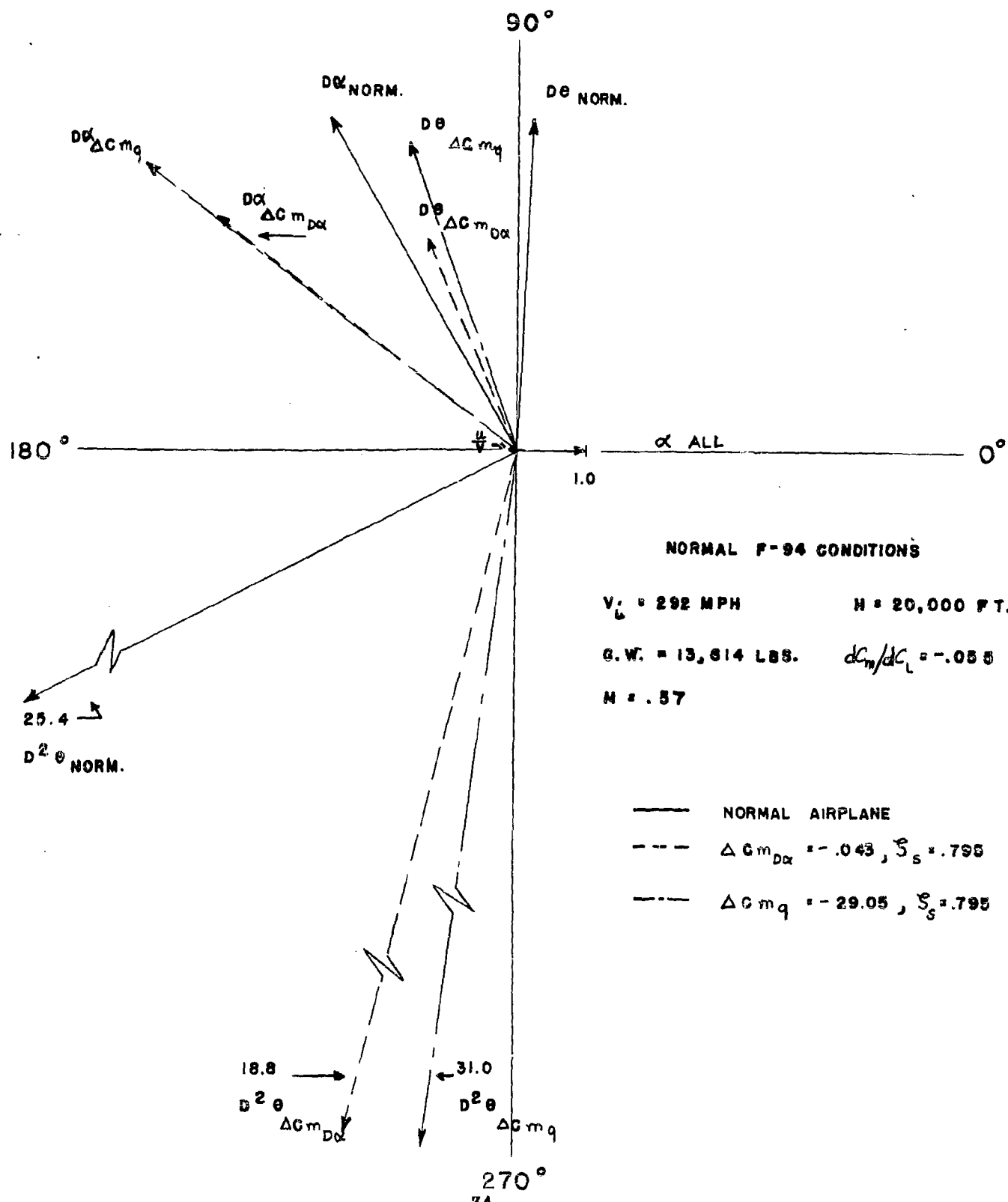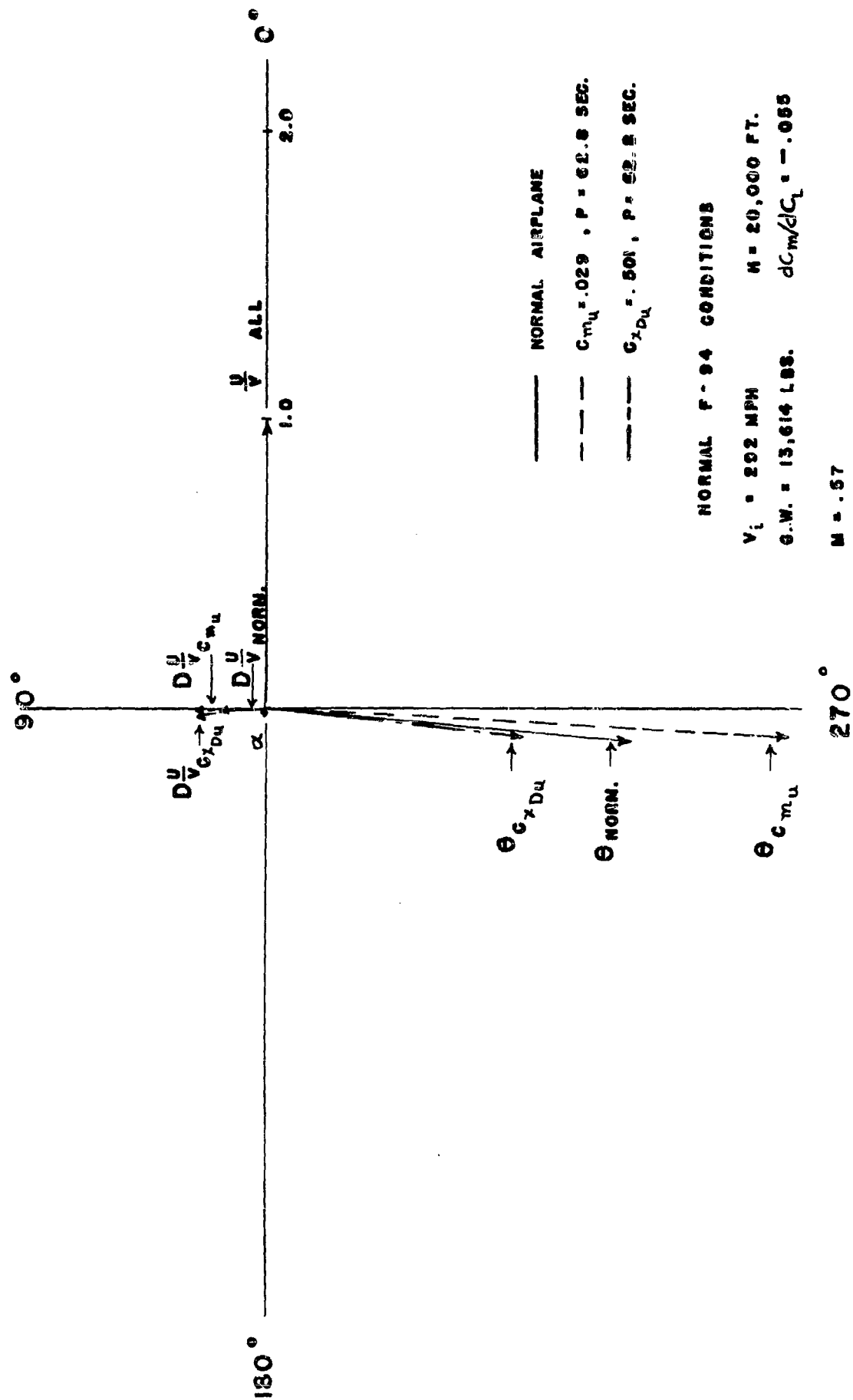
90°

Dα NORM.

Dθ NORM.

Dα ΔC mq

Dθ ΔC mq

Dα ΔC mDα

Dθ ΔC mDα

180°   α ALL   0°

1.0

25.4

D² θ NORM.

NORMAL F-94 CONDITIONS

V_L = 292 MPH   H = 20,000 FT.

G.W. = 13,614 LBS. dC_m/dC_L = -.055

M = .57

——— NORMAL AIRPLANE

- - - $\Delta C_{mD\alpha}$ = -.043, ζ_s = .795

—·—· ΔC_{mq} = -29.05, ζ_s = .795

18.8   31.0

D² θ ΔC mDα   D² θ ΔC mq

270°

FIGURE 10

F - 94

PHASING OF THE PHUGOID

PERIOD VARIED

NORMAL AIRPLANE

$C_{m_u} = .029$, $P = 62.8$ SEC.

$C_{x_{Du}} = .50$, $P = 62.2$ SEC.

NORMAL F - 94 CONDITIONS

$V_i = 292$ MPH

G.W. = 13,614 LBS.

M = .57

H = 20,000 FT.

$dC_m/dC_L = -.055$

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FIGURE 11

F - 94

PHASING OF THE PHUGOID
DAMPING VARIED
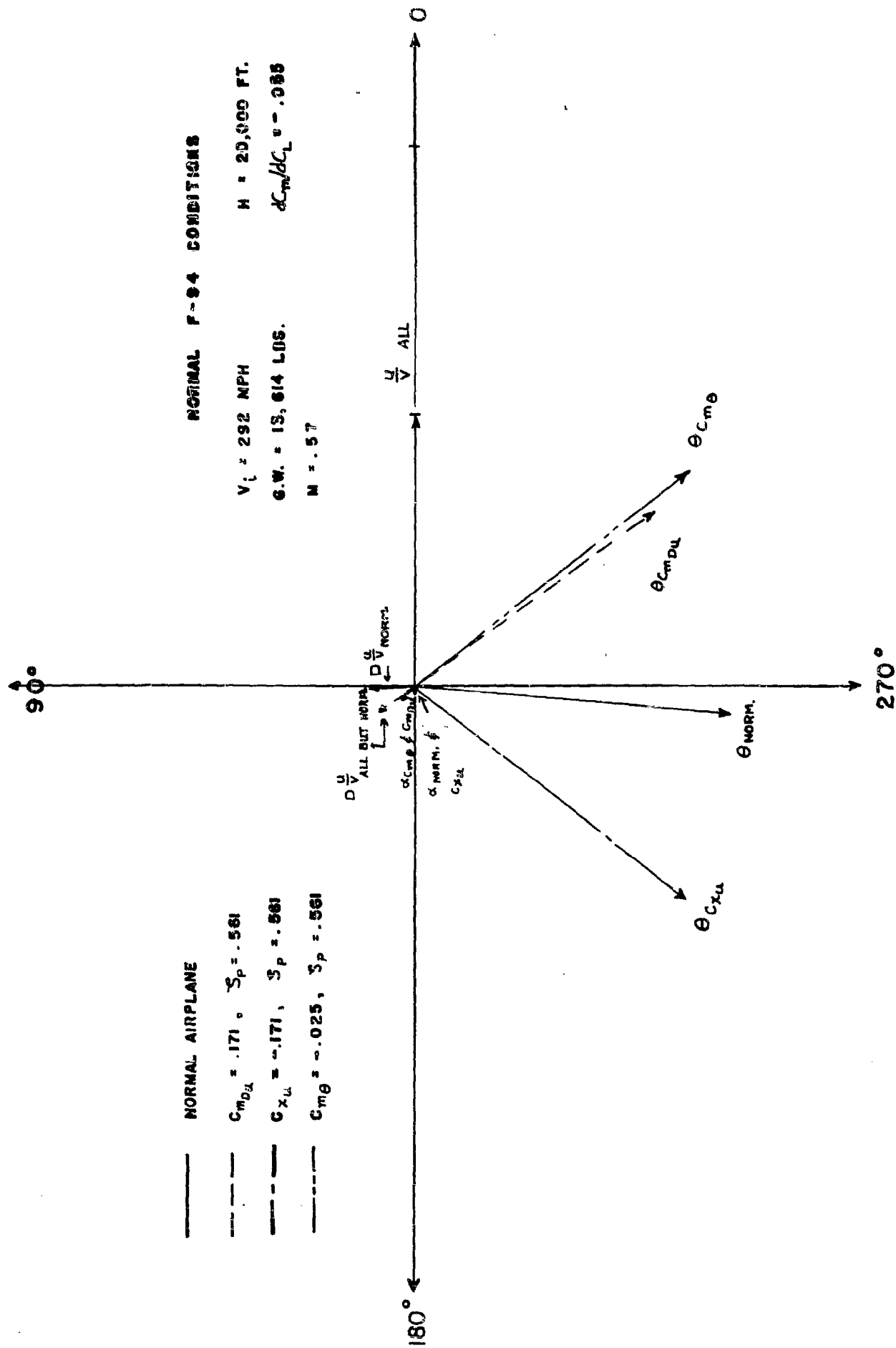
APPENDIX

VECTOR ANALYSIS OF MOTION

A force vector diagram can be drawn for any equation when the direction and amplitude of the variables is known at any natural frequency of the motion. In the aircraft longitudinal motion, the phugoid and short period modes are of greatest interest. The amplitude and phase relationships of the variables will be determined at these frequencies.

<center>Short Period</center>

The short period mode can be represented by the equations:

$$\left(D + \frac{C_{L\alpha}}{2}\right)\alpha - D\theta = 0$$

$$-\frac{\mu}{\kappa_B}\left[C_{m_{D\alpha}}D + C_{m_\alpha}\right]\alpha + \left[D^2 - \frac{C_{m_q}}{\kappa_B}D\right]\theta = 0$$

The natural undamped frequency and damping of the short period can be obtained from Equation (1) and from the relation:

$$\omega_n = \frac{2\pi\,\zeta\,f}{(1 - \zeta_s^2)^{1/2}}$$

the relation between $D\alpha$ and α can be expressed as:

$$D\alpha = \omega_n\,\alpha\,\angle 90° + \epsilon_D$$

where

$$\epsilon_D = \sin^{-1}\zeta_s$$

This expression can be found in Reference (3).

The lift equation can now be used to obtain $D\theta$ in terms of α :

$$D\theta = \left[\omega_n\,\angle 90° + \epsilon_D \quad + \frac{C_{L\alpha}}{2}\right]\alpha$$

A graphical solution will yield sufficiently accurate results. The presence of Du in the short period can be predicted accurately from the drag equation:

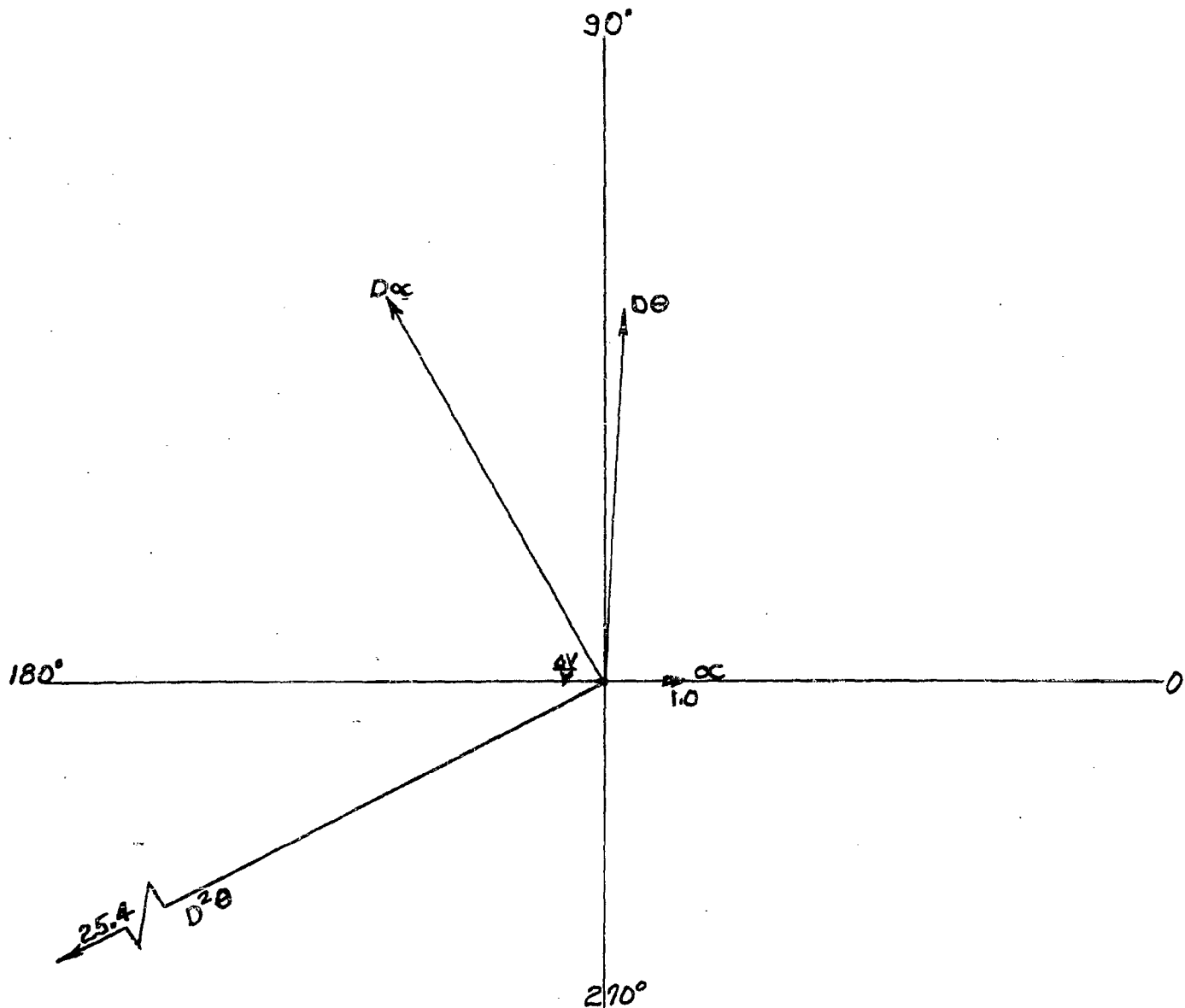$$\left(D + C_D\right)u + \left(\frac{C_{D\alpha}}{2} - C_L\right)\alpha + \frac{C_L}{2}\theta = 0$$

Since C_D is very small compared to Du at the short period frequency, this equation becomes:

$$Du = -\left(\frac{C_{D\alpha}}{2} - C_L\right)\alpha - \frac{C_L}{2}\theta$$

which can be solved graphically.

The quantities $D\theta$ and Du have been determined as vectors in relation to unity α at zero phase. These quantities are shown graphically below for the normal airplane.

<center>- 47 -</center>

SHORT PERIOD PHASING
FREQUENCY VARIED

Phugoid

The phugoid mode can be represented by the three equations of motion:

$$\left(D + C_D\right)u + \left(\frac{C_{D\alpha}}{2} - C_L\right)\alpha + \frac{C_L}{2}\theta = 0$$

$$C_L u + \left(D + \frac{C_{L\alpha}}{2}\right)\alpha - D\theta = 0$$

$$-\frac{\mu}{i_B}\left[C_{mDu}D + C_{mu}\right]u - \frac{\mu}{i_B}\left[C_{mD\alpha}D + C_{m\alpha}\right]\alpha$$

$$+ \left[D^2 - \frac{C_{mq}}{i_B}D\right]\theta = 0$$

If $D^2\Theta$ is neglected, the latter two equations can be combined as:

$$\left[-\frac{\mu}{\iota_B}C_{m_{D\alpha}} - \frac{C_{m_g}}{\iota_B}\right]D\alpha + \left[-\frac{\mu}{\iota_\Theta}C_{m_\alpha} - \frac{C_{m_g}}{\iota_B}\frac{C_{L\alpha}}{2}\right]\alpha =$$

$$\left(\frac{\mu}{\iota_B}C_{m_{Du}}\right)Du + \left[\frac{\mu}{\iota_B}C_{m_u} + \frac{C_L C_{m_g}}{\iota_B}\right]u$$

The vector $D\alpha = \omega_n \alpha\, \underline{/90° + \epsilon_D}$

and $Du = \omega_n u\, \underline{/90° + \epsilon_D}$

The natural undamped phugoid frequency and damping can be obtained from Equation (4) and from:

$$\omega_n = \frac{2\pi\tau}{P(1 - \zeta_P{}^2)^{1/2}}$$

$$\epsilon_D = \sin^{-1}\zeta_P$$

α can then be determined in terms of u. The drag equation can then be used to determine graphically the vector properties of Θ in terms of unit u at zero phase. The normal F-94 phugoid motions are shown vectorially below:

PHASING OF THE PHUGOID
PERIOD VARIED
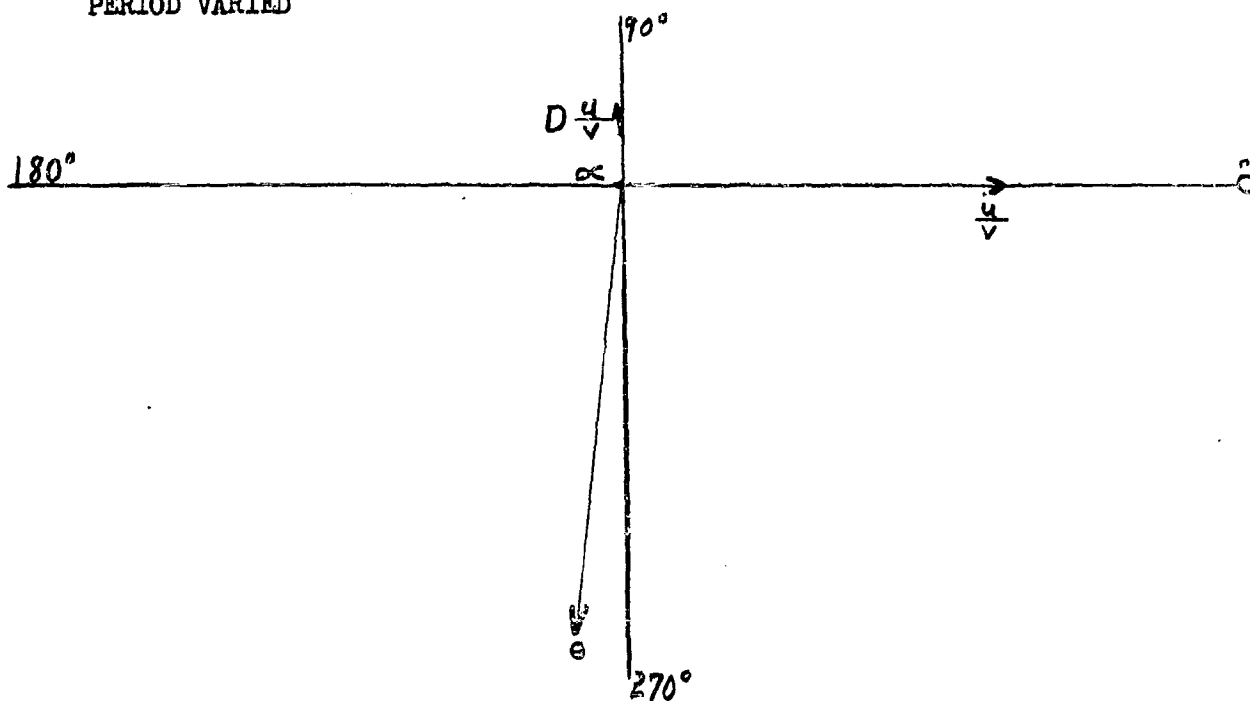
ANALOG COMPUTER SETUP

The longitudinal equations of motion for the F-94 airplane at 400 mph, 20,000 ft. altitude for 5.5% static margin are listed below

$$\Sigma F_x \qquad \frac{du}{dt/\tau_1} = -.049\, u +.122\, \alpha -.275\, \Theta$$

$$\Sigma F_z \qquad \frac{d\alpha}{dt/\tau_1} = -.55\, u -6.14\, \alpha + \frac{d\Theta}{dt/\tau_1}$$

$$\Sigma M_y \qquad \frac{d^2\Theta}{d(t/\tau_1)^2} = -107\, \alpha -1.60\, \frac{d\alpha}{dt/\tau_1} -3.5\, \frac{d\Theta}{dt/\tau_1} -311\, \delta_e$$

where $\tau_1 = 5.0$ sec.

The elevator was positioned by the signals:

$$\delta_e = \delta_{input} + .412\, \alpha + .0219\, \frac{d\alpha}{dt/\tau_1} -.0307\, u -.0875\, \frac{du}{dt/\tau_1}$$

applying $\Delta C_{m_\alpha} = -.389$, $\Delta C_{m_{D\alpha}} = -.043$, $C_{m_u} = .029$, $C_{m_{Du}} = .171$

Inputs

A forward velocity gust is applied to the airplane on the analog by entering an initial condition on u. This is equivalent to a forward velocity increment of the air relative to the ground. Since the plane's velocity with respect to the ground will not change, this gust will give a sudden increase in the velocity of the plane with respect to the air as seen mathematically below:

$$u_{pa} = u_{pg} + u_{ag}$$

This u_{ag} provided the disturbance to excite the phugoid oscillation.

It should be noted that a sudden increase in u_{pa} will cause an impulse in the rate of change of airspeed as measured by a pitot tube. Thus, as $C_{m_{Du}}$ is provided by sensing the rate of change of u_{pa}, an impulse in pitching moment would be applied to the airplane. The auxiliary surface is limited to 2 deg/sec., however, so that the rate of change of pitching moment is drastically limited.

In order to examine the response of the airplane without this velocity limiter, initial conditions on u and Θ were entered on the analog. The condition on Θ arises from the previously noted effect of $C_{m_{Du}}$. Thus, the integral of the moment equation must be in balance before the gust is applied at time -0 and just after application at time +0. This yields,

$$\int_{t_0}^{+o} \frac{d^2\theta}{d(t/\tau_1)^2} \, dt/\tau_1 = \int_{-o}^{+o} -311 \cdot \left[-.0875\right] \frac{d\,u}{dt/\tau_1} \, dt/\tau_1$$

or

$$\left[\frac{d\,\theta}{dt/\tau_1}\right]_o = 27.2 \; u_o$$

This relation between $\overset{\circ}{\theta}_o$ and u_o is necessary to represent $u_{a/g}$ gust properly when $C_{m_{Du}}$ of .171 is present.

An angle of attack gust or a vertical velocity gust will be represented similarly by initial conditions on $\propto$ and $\overset{\circ}{\theta}$. Thus, for the normal airplane:

$$\left[\frac{d\,\theta}{dt/\tau_1}\right]_o = -1.60 \; \propto_o$$

When $\triangle C_{m_{D\propto}}$ is present, an $\propto_o$ gust creates a larger $\overset{\circ}{\theta}_o$ as:

if $\triangle C_{m_{D\propto}} = -.043;$ $\quad (d\,\theta/dt/\tau_1)_o = -8.40 \; \propto_o$

Besides the inputs of initial conditions on u, $\overset{\circ}{\theta}$ and $\propto$, $\overset{\circ}{\theta}$; the airplane response to a step elevator and to a ramp elevator was examined. The response to a ramp represented the most severe response possible with the velocity limiter used on the auxiliary surface. It should be noted that although the equations are set up for elevator inputs, auxiliary surface inputs can be obtained by ratioing the response in terms of $C_{m_{\delta a}}/C_{m_{\delta e}}$.

Wiring Diagram

The wiring diagram is included next to indicate the circuitry necessary to obtain the responses:

ANALOG WIRING DIAGRAM
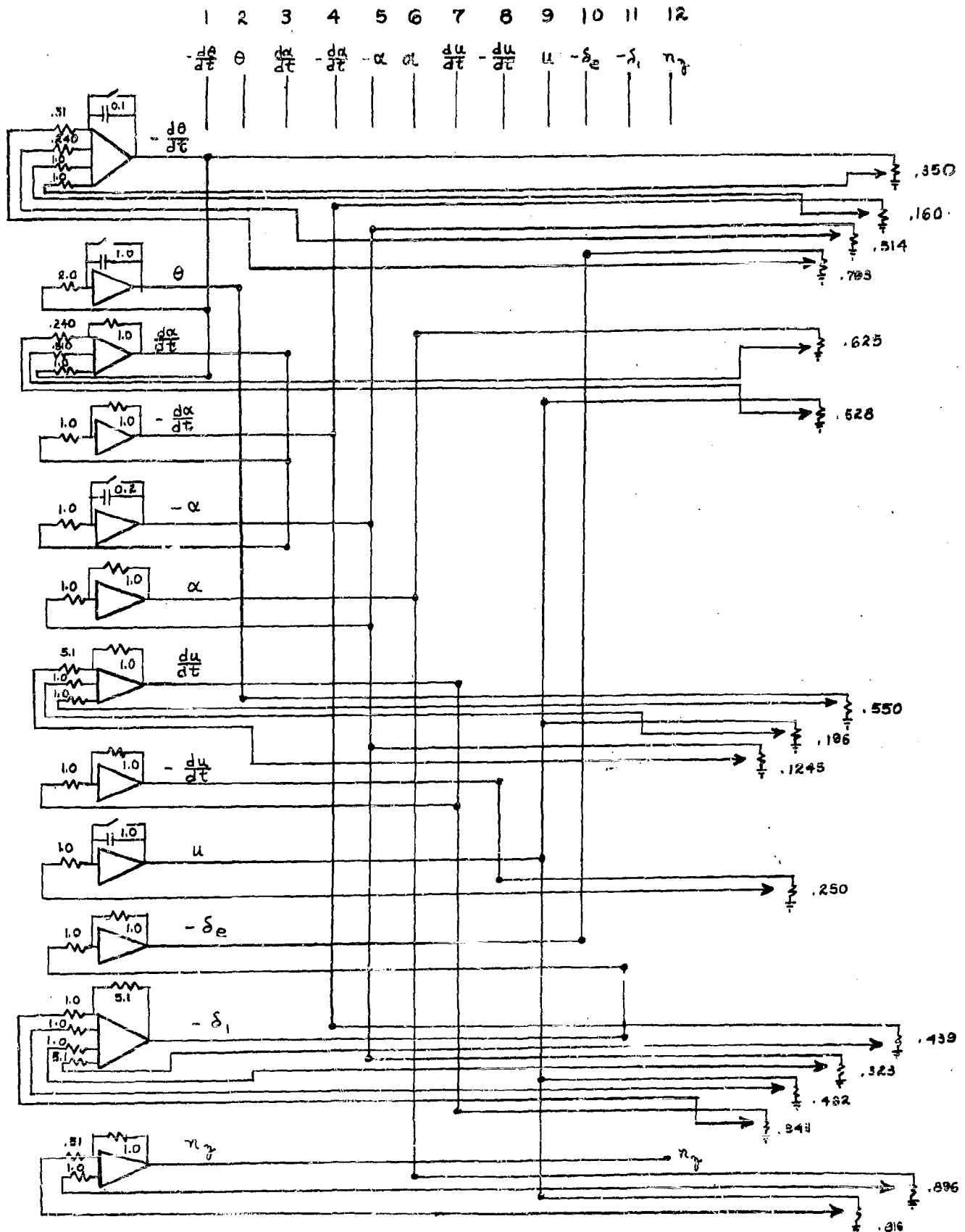
STEADY STATE VALUES

The following formulae are listed to aid in determining the steady state values of the aircraft motions following a step elevator deflection:

$$\Delta V_L = \frac{V_i C_{m_{S_e}}}{2 C_L} \cdot \frac{S_{e_{SS}}}{\left(\frac{d C_m}{d C_L} - \frac{C_{m_u}}{2 C_L}\right)}$$

$$\alpha = -\frac{C_{m_s} S_{e_{SS}}}{C_{L_\alpha}} \cdot \frac{1}{\left(\frac{d C_m}{d C_L} - \frac{C_{m_u}}{2 C_L}\right)}$$

$$\Theta = -\frac{C_{m_{S_e}} S_{e_{SS}}}{C_L} \cdot \frac{\left(\frac{C_D}{C_L} - \frac{C_{D_K}}{C_{L_K}} + \frac{C_L}{C_{L_K}}\right)}{\left(\frac{d C_m}{d C_L} - \frac{C_{m_u}}{2 C_L}\right)} \qquad \text{(assuming } C_{L_S} = 0\text{)}$$

The steady state values in the short period mode of motion are:

$$\alpha = \frac{C_{m_{S_e}} S_{e_S}}{C_{L_\alpha}} \cdot \frac{1}{\left(-\frac{d C_m}{d C_L} - \frac{C_{m_q}}{2 \mu}\right)}$$

$$\frac{d \Theta}{d t} = \frac{C_{m_{S_e}} S_{e_S}}{2 \tau} \cdot \frac{1}{\left(-\frac{d C_m}{d C_L} - \frac{C_{m_q}}{2 \mu}\right)}$$

The maximum value of α in the short period, and of ΔV_L and Θ in the phugoid can be computed from:

$$\chi \text{ max.} = \chi \text{ steady state } \left(1 + \frac{\% \text{ overshoot}}{100}\right)$$

where either α, V_L, or Θ can be represented by χ
The percentage overshoot is a function of ζ along and is shown below:

ζ — damping ratio (% critical)